Optimization in modern power systems

Lecture 7: AC-OPF

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The Goals for Today!

- Review of Day 6
- Questions and Clarifications on Assignments
- AC-OPF
- Y_{bus} and Y_{line}
- From the AC to the DC power flow equations
- Mid-term Course Evaluation/Feedback
- Assignment 3
- Multi-objective Optimization

Reviewing Day 6 in Groups!

- For 10 minutes discuss with the person sitting next to you about:
  - Three main points we discussed about yesterday's review
  - One topic or concept that is not so clear to you and you would like to hear again about it

Points you would like to discuss?

Questions about the Assignments?

AC-OPF

- Minimize

- subject to:

AC-OPF

- Minimize

  Costs, Line Losses, other?

- subject to:

AC-OPF

- Minimize

Costs, Line Losses, other?

- subject to:

AC Power Flow equations

Line Flow Constraints

Generator Active Power Limits

Generator Reactive Power Limits

Voltage Magnitude Limits

(Voltage Angle limits to improve solvability)

(maybe other equipment constraints)

AC-OPF

- Minimize

Costs, Line Losses, other?

- subject to:

AC Power Flow equations

Line Flow Constraints — Line Current Limits / Apparent Power Flow limits / Active Power Flow limits

Generator Active Power Limits

Generator Reactive Power Limits

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(maybe other equipment constraints)

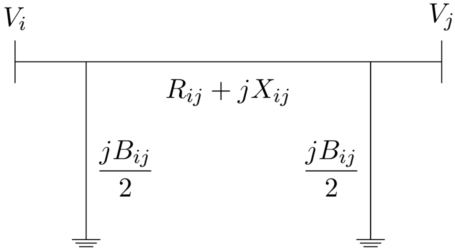
- Optimization vector: $[P\ Q\ V\ \theta]^T$

AC-OPF[1]

obj.function	$\min c^T P_G$
AC flow	$S_G - S_L = diag(\overline{V})\overline{Y}^*_{\text{bus}}\overline{V}^*$
Line Current	$\lvert\overline{Y}_{\text{line},i\rightarrow j}\overline{V}\rvert \leq I_{line,max}$
	$\lvert\overline{Y}_{\text{line},j\rightarrow i}\overline{V}\rvert \leq I_{line,max}$
or Apparent Flow	$\lvert\overline{V}_i\overline{Y}^*_{\text{line},i\rightarrow j,\text{i-row}}\overline{V}^*\rvert \leq S_{i\rightarrow j,max}$
	$\lvert\overline{V}_j\overline{Y}^*_{\text{line},j\rightarrow i,\text{j-row}}\overline{V}^*\rvert \leq S_{j\rightarrow i,max}$
Gen. Active Power	$0 \leq P_G \leq P_{G,max}$
Gen. Reactive Power	$-Q_{G,max} \leq Q_G \leq Q_{G,max}$
Voltage Magnituge	$V_{min} \leq V \leq V_{max}$
Voltage Magnituge	$V_{min} \leq V \leq V_{max}$
Voltage Angle	$\theta_{min} \leq \theta \leq \theta_{max}$

[1] All shown variables are vectors or matrices. The bar above a variable denotes complex numbers. $(\cdot)^*$ denotes the complex conjugate. To simplify notation, the bar denoting a complex number is dropped in the following slides. Attention! The *current* flow constraints are defined as *vectors*, i.e. for all lines. The apparent power *line* constraints are defined *per line*.

Current flow along a line



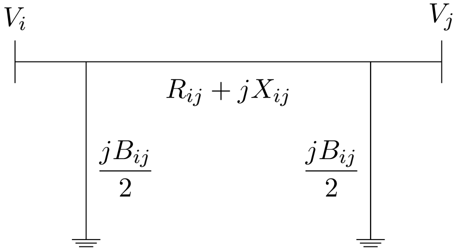
π-model of the line

It is:

$$y_{ij} = \frac{1}{R_{ij} + jX_{ij}}$$

$y_{sh,i} = j\frac{B_{ij}}{2} +$ other shunt elements connected to that bus

Current flow along a line



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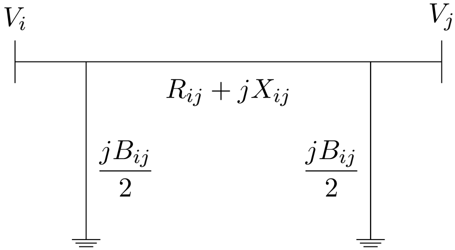
$$i \rightarrow j: \quad I_{i\rightarrow j} = y_{sh,i}V_i + y_{ij}(V_i - V_j) \Rightarrow I_{i\rightarrow j} = \begin{bmatrix} y_{sh,i} + y_{ij} & -y_{ij} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix}$$

Current flow along a line



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$$j \rightarrow i: \quad I_{j\rightarrow i} = y_{sh,j}V_j + y_{ij}(V_j - V_i) \Rightarrow I_{j\rightarrow i} = \begin{bmatrix} -y_{ij} & y_{sh,j} + y_{ij} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix}$$

Line Admittance Matrix Y_{line}

- $\mathbf{Y}_{\text{line}}$ is an $L \times N$ matrix, where L is the number of lines and N is the number of nodes

- if row k corresponds to line $i - j$:
  - $Y_{\text{line},ki} = y_{sh,i} + y_{ij}$
  - $Y_{\text{line},kj} = -y_{ij}$

- $y_{ij} = \dfrac{1}{R_{ij} + jX_{ij}}$ is the admittance of line ij
- $y_{sh,i}$ is the shunt capacitance $jB_{ij}/2$ of the π-model of the line
- We must create two Y_{line} matrices. One for $i \to j$ and one for $j \to i$

Bus Admittance Matrix $Y_{\textsf{bus}}$

$$S_i = V_i I_i^*$$

$$I_i = \sum_k I_{ik}, \text{ where } k \text{ are all the buses connected to bus } i$$

Example: Assume there is a line between nodes $i-m$, and $i-n$. It is:

$$\begin{aligned} I_i &= I_{im} + I_{in} \\ &= (y_{sh,i}^{i\to m} + y_{im})V_i - y_{im}V_m + (y_{sh,i}^{i\to n} + y_{in})V_i - y_{in}V_n \\ &= (y_{sh,i}^{i\to m} + y_{im} + y_{sh,i}^{i\to n} + y_{in})V_i - y_{im}V_m - y_{in}V_n \end{aligned}$$

$$I_i = [\underbrace{y_{sh,im} + y_{im} + y_{sh,in} + y_{in}}_{Y_{\textsf{bus},ii}} \; \underbrace{-y_{im}}_{Y_{\textsf{bus},im}} \; \underbrace{-y_{in}}_{Y_{\textsf{bus},in}}][V_i \; V_m \; V_n]^T$$

Bus Admittance Matrix Y_{bus}

- $\mathbf{Y}_{\text{bus}}$ is an $N \times N$ matrix, where N is the number of nodes
- diagonal elements: $Y_{\text{bus},ii} = y_{sh,i} + \sum_k y_{ik}$, where k are all the buses connected to bus i
- off-diagonal elements:
  - $Y_{\text{bus},ij} = -y_{ij}$ if nodes i and j are connected by a line[2]
  - $Y_{\text{bus},ij} = 0$ if nodes i and j are not connected
- $y_{ij} = \dfrac{1}{R_{ij} + jX_{ij}}$ is the admittance of line ij
- $y_{sh,i}$ are all shunt elements connected to bus i, including the shunt capacitance of the π-model of the line

[2] If there are more than one lines connecting the same nodes, then they must all be added to $Y_{\text{bus},ij}, Y_{\text{bus},ii}, Y_{\text{bus},jj}$.

AC Power Flow Equations

$$
\begin{aligned}
S_i &= V_i I_i^* \\
&= V_i Y_{\text{bus}}^* V^*
\end{aligned}
$$

For all buses $S = [S_1 \dots S_N]^T$:

$$S_{\text{gen}} - S_{\text{load}} = diag(V) Y_{\text{bus}}^* V^*$$

From AC to DC Power Flow Equations

- The power flow along a line is:

$$S_{ij} = V_i I_{ij}^* = V_i(y_{sh,i}^* V_i^* + y_{ij}^*(V_i^* - V_j^*))$$

- Assume a negligible shunt conductance: $g_{sh,ij} = 0 \Rightarrow y_{sh,i} = jb_{sh,i}$.
- Given that $R << X$ in transmission systems, for the DC power flow we assume that $z_{ij} = r_{ij} + jx_{ij} \approx jx_{ij}$. Then $y_{ij} = -j\dfrac{1}{x_{ij}}$.
- Assume: $V_i = V_i\angle 0$ and $V_j = V_j\angle\delta$, with $\delta = \theta_j - \theta_i$.

$$\begin{aligned} I_{ij}^* &= -jb_{sh,i}V_i + j\frac{1}{x_{ij}}(V_i - (V_j\cos\delta - jV_j\sin\delta)) \\ &= -jb_{sh,i}V_i + j\frac{1}{x_{ij}}V_i - j\frac{1}{x_{ij}}V_j\cos\delta - \frac{1}{x_{ij}}V_j\sin\delta \end{aligned}$$

From AC to DC Power Flow Equations (cont.)

- Since V_i is a real number, it is:

$$P_{ij} = \Re\{S_{ij}\} = V_i \Re\{I_{ij}^*\} = -\frac{1}{x_{ij}} V_i V_j \sin\delta$$

- With $\delta = \theta_j - \theta_i$, it is:

$$P_{ij} = \frac{1}{x_{ij}} V_i V_j \sin(\theta_i - \theta_j)$$

- We further make the assumptions that:
  - V_i, V_j are constant and equal to 1 p.u.
  - $\sin\theta \approx \theta$, θ must be in rad

Then

$$P_{ij} = \frac{1}{x_{ij}} (\theta_i - \theta_j)$$

Mid-term evaluation/feedback

Assignment 3

- Implement an AC-OPF with the objective to minimize the active power generation costs
- Compare the Power Dispatch between the AC-OPF and the DC-OPF for the same system
- Compare the Nodal Prices between the AC-OPF and the DC-OPF for the same system
- Bonus question: AC-OPF with the objective to minimize line losses

Multi-objective Optimization

- In an optimization we can have several different objectives that we want to minimize at the same time
- Example or OPF: minimize active power costs *and* minimize line losses

$$\min w_1 F_1(x) + w_2 F_2(x) \tag{1}$$

Multi-objective Optimization (cont.)

- The weights w_1, w_2 are used for two reasons:
  1. Show a preference of which objective is more important. Higher weights mean that this part of the objective function is more important to get minimized.
  2. Brin the numerical values of $F_1(x)$ and $F_2(x)$ to the same order of magnitude. E.g if $F_1(x)$ represents costs of millions of DKK, while $F_2(x)$ represents line losses of MWs, we would have $F_1(x) + F_2(x) \approx 1'000'000 + 50$. The solver sees this sum as one value. As a result, the solver will prefer to save 100 DKK than reduce the losses from 50 MW or 10 MW. Appropriate weights help us to avoid this!

- We must be very careful when we include competing objectives in the objective function:
  - the optimization will not find the minimum of any of the objectives
  - the obtained solution will be heavily dependent on the weights w_1, w_2