



# Implicit layers: A toolkit for AI in power systems

Priya L. Donti

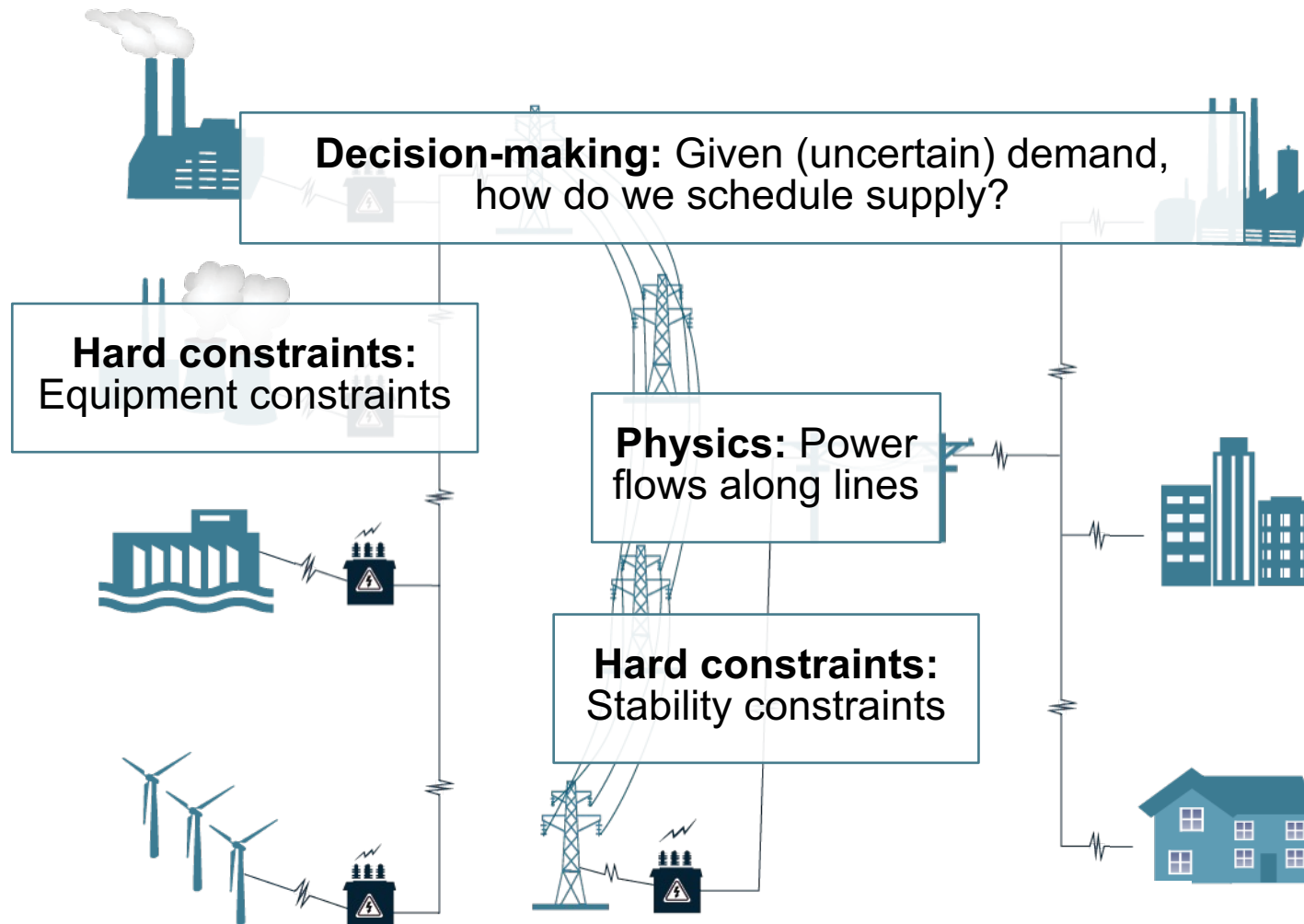
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# Power systems problems involve physics, hard constraints, and decision-making



## Trad. optimization & control

- Satisfies (many) constraints
- Struggles with speed / scale



## Machine learning (ML)

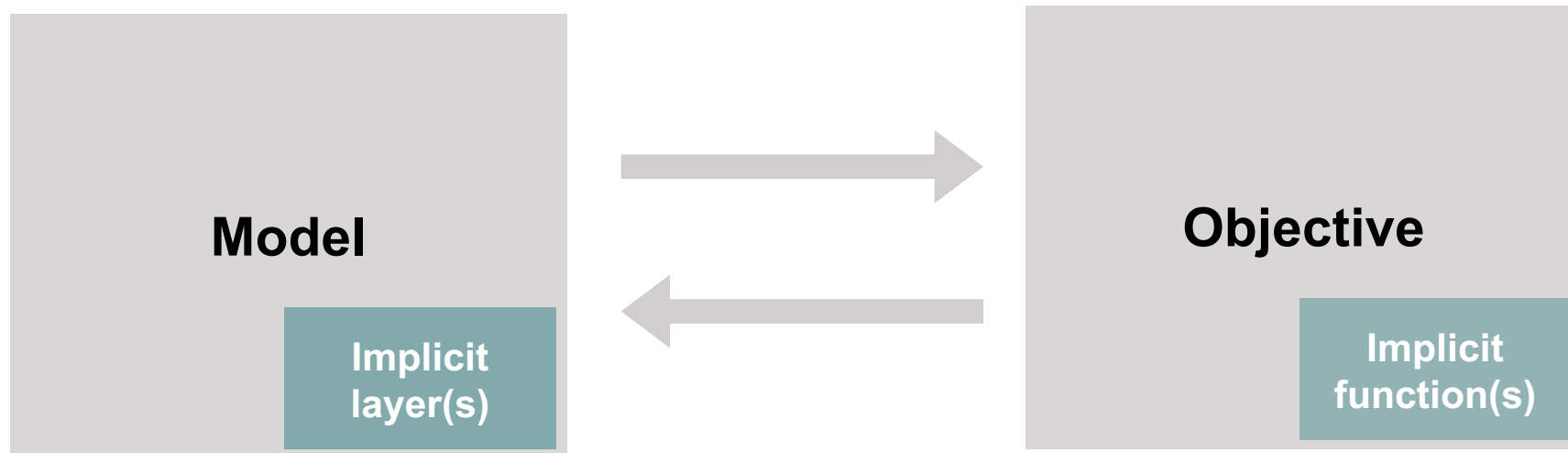
- Fast and scalable
- Struggles with constraints

Figure adapted from: US Congressional Budget Office



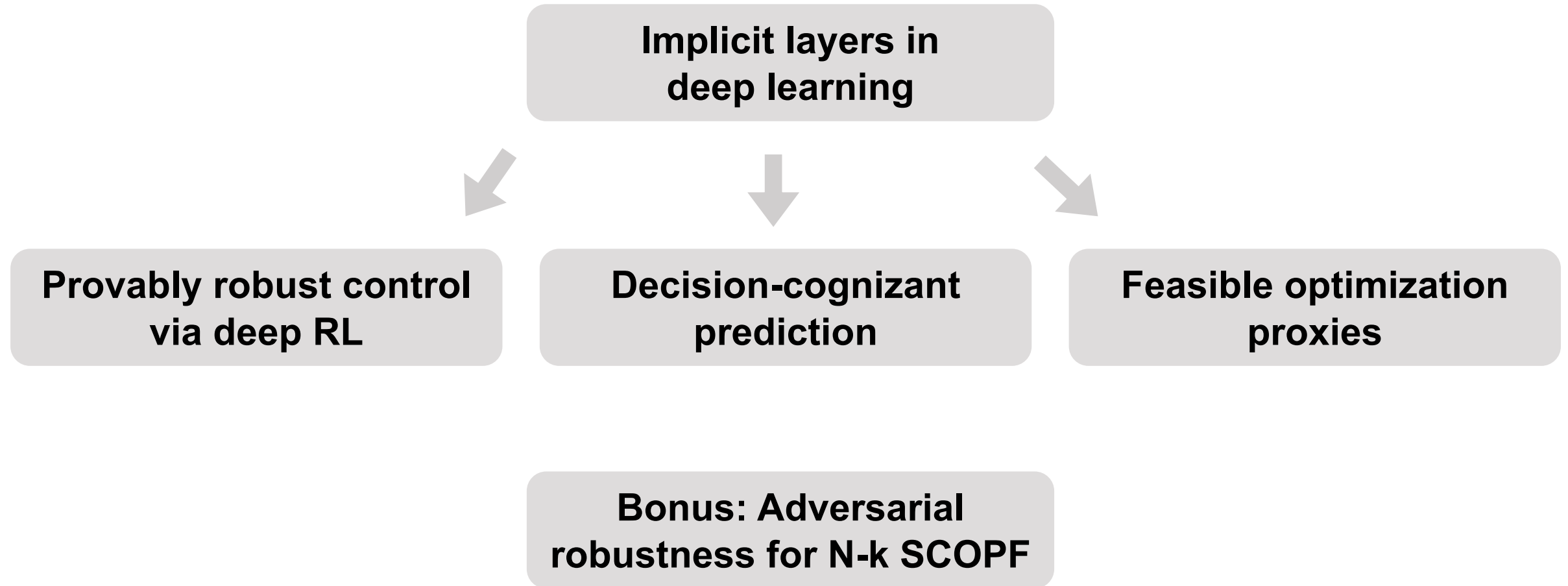
# Implicit layers in deep learning

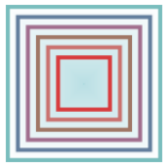
Toolkit for developing deep learning methods that incorporate knowledge of physics, hard constraints, or downstream decision-making procedures



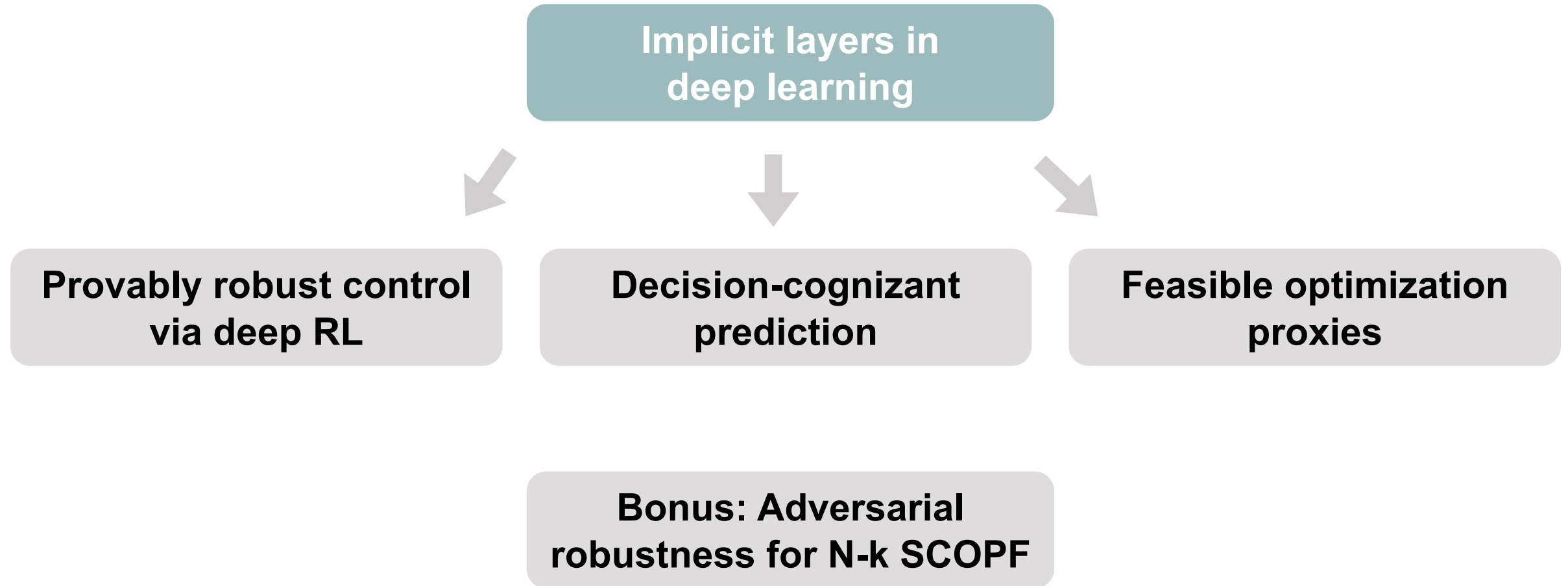


# Talk outline



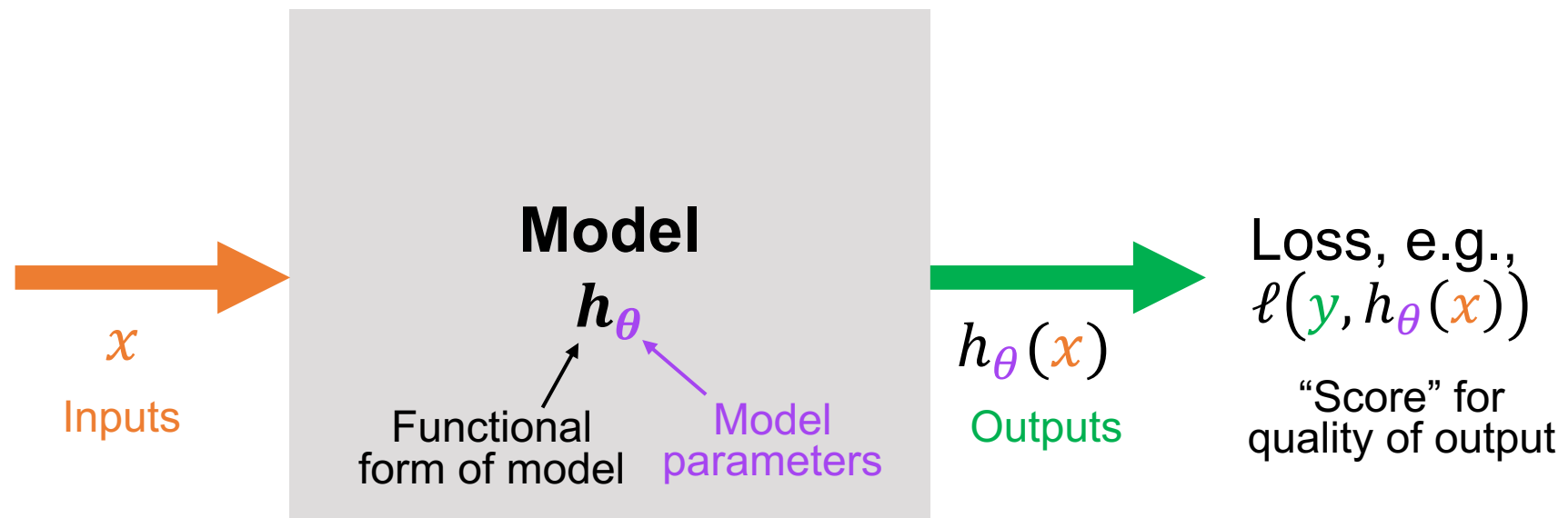


# Talk outline





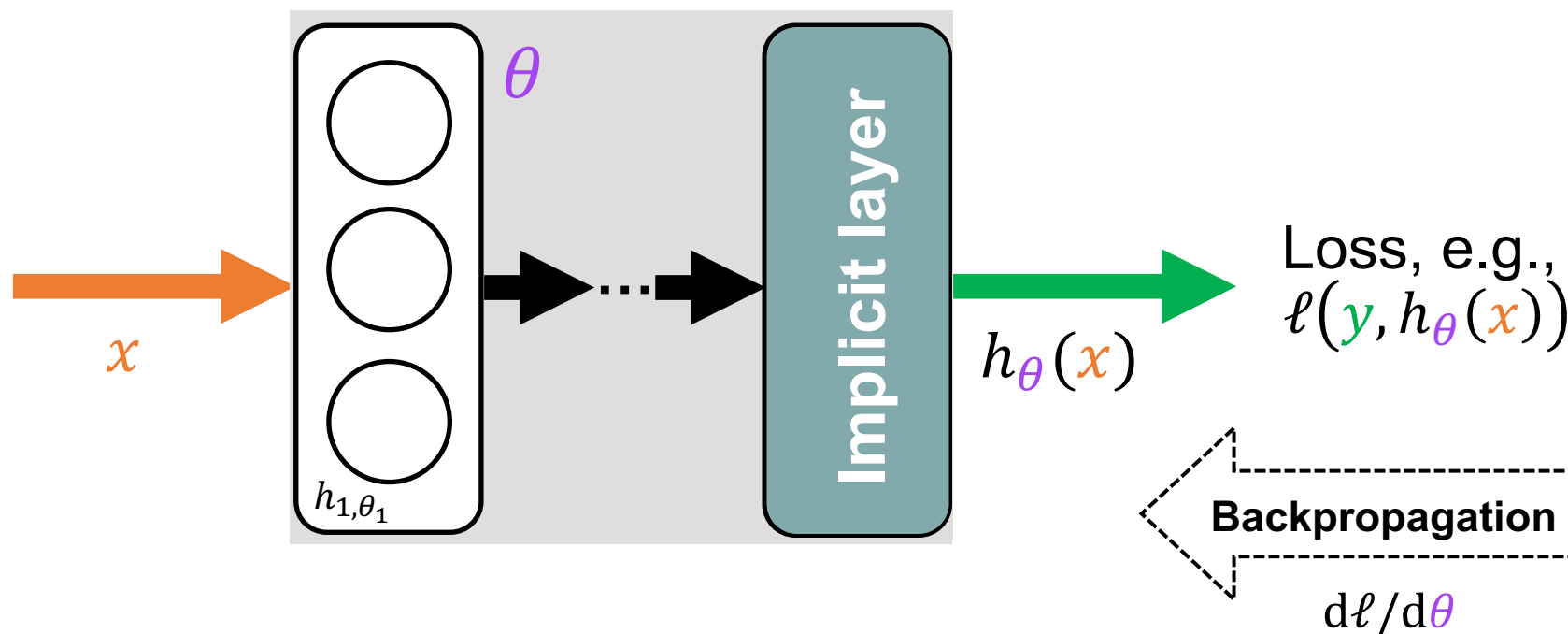
# Deep learning is differentiable function composition





# Deep learning is differentiable function composition

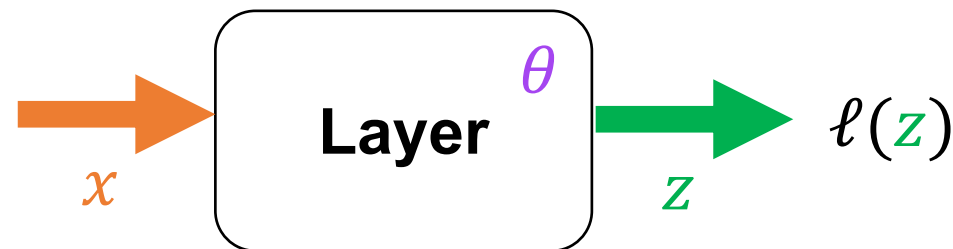
- Neural network  $h_{\theta}$  = composition of nonlinear, parameterized functions (*layers*)
- Update parameters  $\theta$  to minimize loss  $\ell$  using gradients from *backpropagation*
- All components (layers and loss) **must be differentiable**



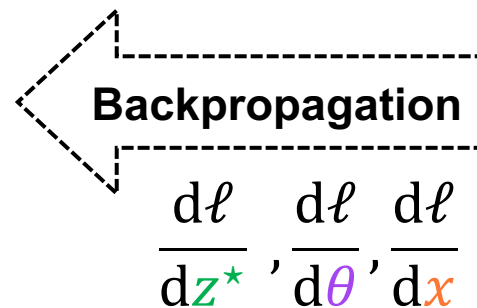


# Backpropagation and gradient descent (one layer)

Forward pass:



Backward pass:



via chain rule

$$\frac{d\ell}{d\theta} = \frac{d\ell}{dz^*} \frac{dz^*}{d\theta}$$

$$\frac{d\ell}{dx} = \frac{d\ell}{dz^*} \frac{dz^*}{dx}$$

Update (gradient descent):

$$\theta \leftarrow \theta - \alpha \frac{d\ell}{d\theta}$$



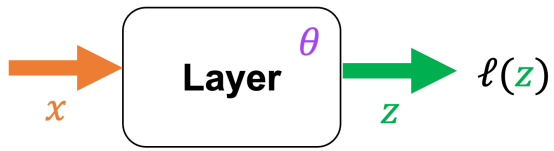


# Explicit vs. implicit layer

## Explicit layer

## Implicit layer

### Forward pass:



$$z = f(x, \theta)$$

$$[\text{e.g., } z = \sigma(\theta^T x + \theta_0)]$$

Find  $z$  such that

$$g(z, x, \theta) = 0$$

[e.g., power flow]

### Backward pass:

$$\frac{d\ell}{d\theta} = \frac{d\ell}{dz^*} \frac{dz^*}{d\theta} \quad \frac{d\ell}{dx} = \frac{d\ell}{dz^*} \frac{dz^*}{dx}$$

$$\frac{dz^*}{dx} = \frac{df(x, \theta)}{dx}$$

Find  $dz^*/dx$  such that

$$\frac{dg(z^*, x, \theta)}{dx} = 0$$

by using implicit function theorem at a solution point



# Example: Differentiable quadratic programming layer

**Insight:** Apply the implicit function theorem to the KKT optimality conditions

QP layer (output  $z$ , all else are inputs/params)

$$\begin{aligned} &\underset{z}{\text{minimize}} && \frac{1}{2} z^T Q z + q^T z \\ &\text{subject to} && A z = b \\ &&& G z \leq h \end{aligned}$$

Selected KKT optimality conditions

$$\begin{aligned} Q z^* + q + A^T v^* + G^T \lambda^* &= 0 \\ A z^* - b &= 0 \\ \text{diag}(\lambda^*)(G z^* - h) &= 0 \end{aligned}$$

**Step 1:** Apply implicit function theorem to the KKT conditions

$$\underbrace{\begin{bmatrix} Q & G^T & A^T \\ \text{diag}(\lambda^*)G & \text{diag}(G z^* - h) & 0 \\ A & 0 & 0 \end{bmatrix}}_{\text{Generalized Jacobian of KKT conditions}} \underbrace{\begin{bmatrix} dz \\ d\lambda \\ dv \end{bmatrix}}_{\text{Desired gradients}} = - \underbrace{\begin{bmatrix} dQ z^* + dq + dG^T \lambda^* + dA^T v^* \\ \text{diag}(\lambda^*)dG z^* - \text{diag}(\lambda^*)dh \\ dA z^* - db \end{bmatrix}}_{\text{Gradients of problem parameters}}$$

**Step 2:** Use “Jacobian-vector trick” for efficient backpropagation

Brandon Amos and J. Zico Kolter. “OptNet: Differentiable optimization as a layer in neural networks.” *ICML 2017*.  
 Priya L. Donti, Brandon Amos, and J. Zico Kolter. “Task-based end-to-end model learning in stochastic optimization.” *NeurIPS 2017*.



# Many types of implicit layers

**Insight:** Apply implicit function theorem to equilibrium or optimality conditions (and use computational tricks to efficiently compute  $d\ell/d\theta$  directly)

**OptNet: Differentiable Optimization as a Layer in Neural Networks**

**Task-based End-to-end Model Learning  
in Stochastic Optimization**

**SATNet: Bridging deep learning and logical reasoning using a differentiable**

**Powerful toolkit for incorporating important  
structure into deep learning methods**

**Maximization**

**End-to-End Differentiable Physics  
for Learning and Control**

**Differentiable Convex Optimization Layers**

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**Neural Ordinary Differential Equations**

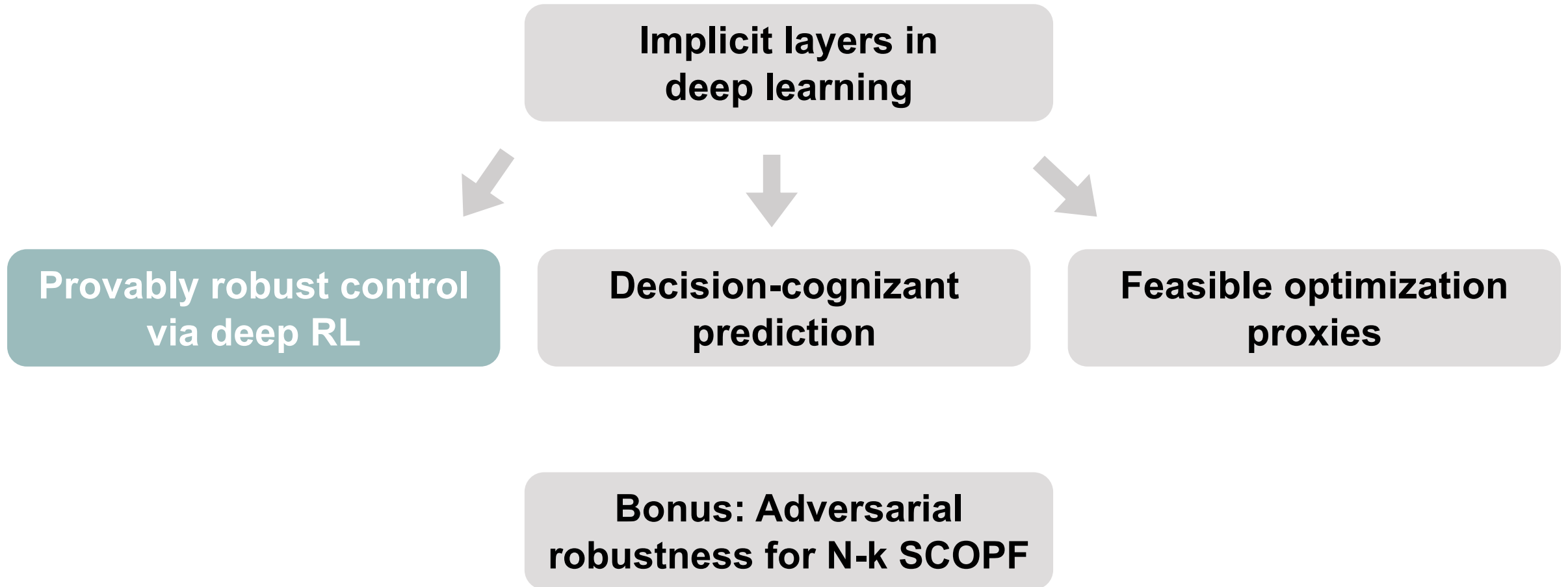
**Ricky T. Q. Chen\*, Yulia Rubanova\*, Jesse Bettencourt\*, David Duvenaud**  
University of Toronto, Vector Institute  
{rtqichen, rubanova, jessebett, duvenaud}@cs.toronto.edu

**Kelsey B. Allen**

**Joshua B. Tenenbaum**



# Talk outline



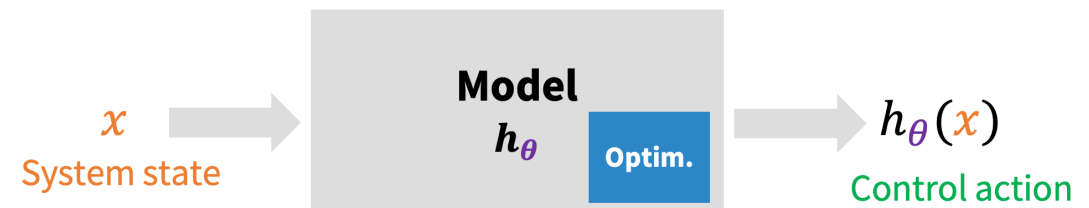


# Overview: Provably robust control via RL



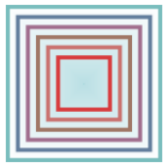
**Motivation:** Need for well-performing control methods that also guarantee enforcement of hard constraints

**Approach:** Use implicit layers in deep reinforcement learning (RL) to guarantee enforcement of hard constraints

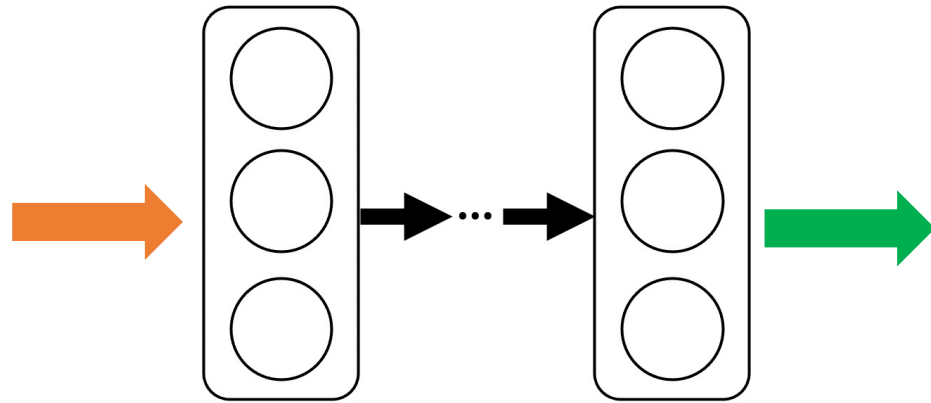


## Settings:

- Asymptotic stability in power grids
- Realistic-scale building control



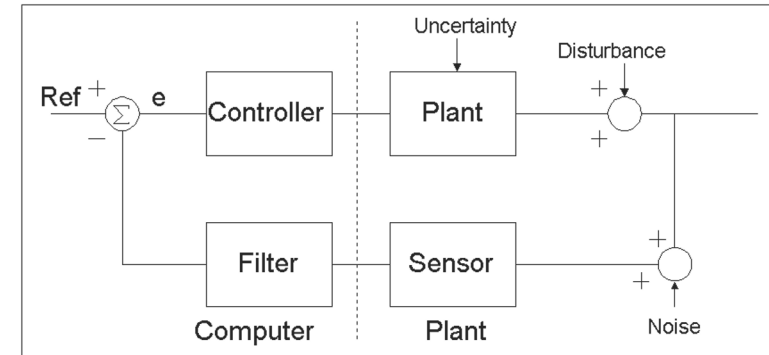
# Deep reinforcement learning vs. robust control



## Deep RL

**Pro:** Expressive, well-performing policies

**Con:** Potential (catastrophic) failures



## Robust control

**Pro:** Provable stability guarantees

**Con:** Simple policies (e.g., linear)

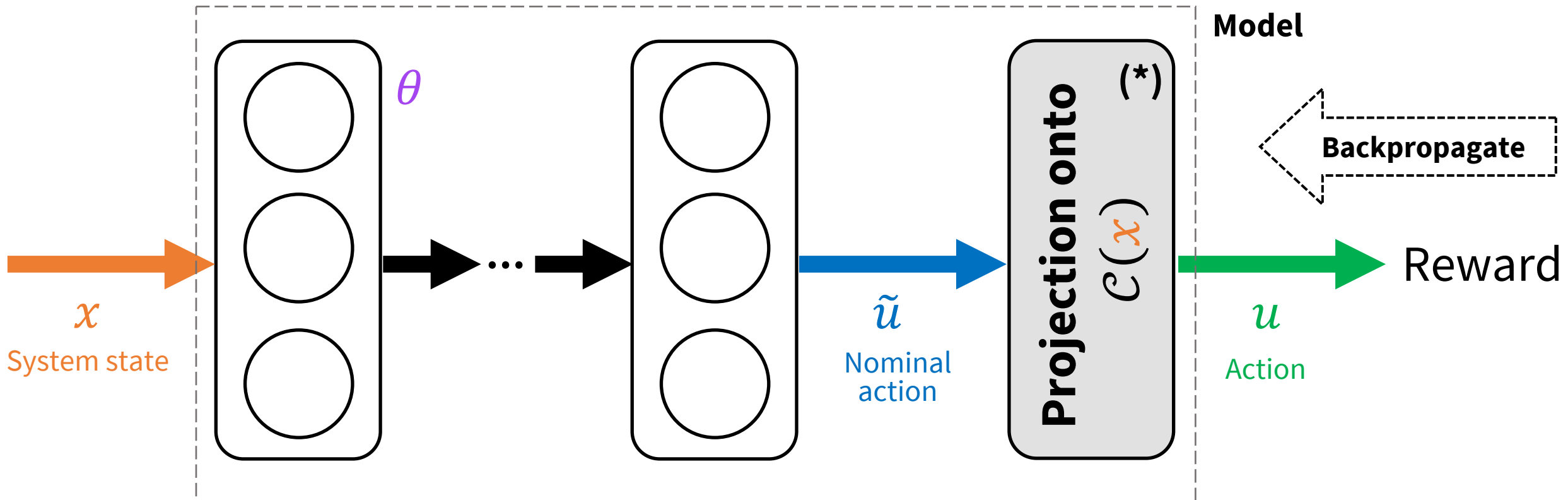
Can we improve performance while still guaranteeing stability?

Priya L. Donti, Melrose Roderick, Mahyar Fazlyab, and J. Zico Kolter. "Enforcing robust control guarantees within neural network policies." *International Conference on Learning Representations (ICLR) 2021*.



# Differentiable projection onto stabilizing actions

Deep learning-based policy with **provable robustness guarantees** (even for a randomly initialized neural network), trainable using reinforcement learning





## Finding a set of stabilizing actions (example)

**Insight:** Find a set of actions that are guaranteed to satisfy relevant Lyapunov stability criteria at a given state, even under worst-case conditions

**Given the following** (from robust control):

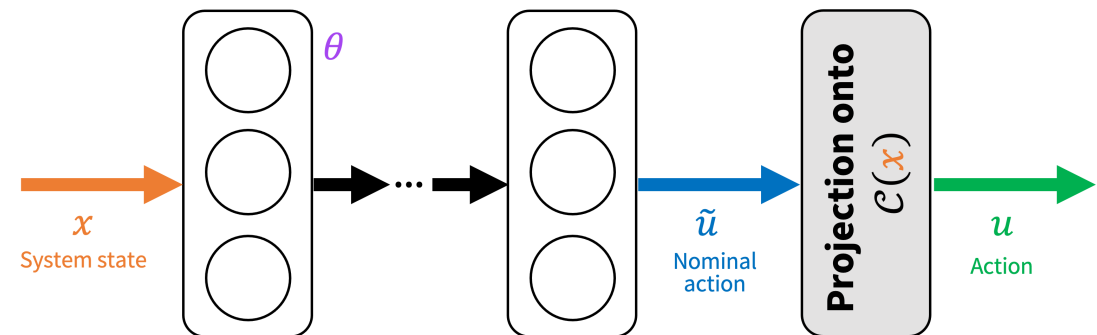
- Uncertainty model: e.g.,  $\dot{x}(t) \in Ax(t) + Bu(t) + Gw(t)$  s.t.  $\|w(t)\|_2 \leq \|Cx(t) + Du(t)\|_2$
- Lyapunov function  $V$  obtained via robust control synthesis
- Exponential stability criterion:  $\dot{V}(x(t)) \leq -\alpha V(x(t)), \forall x \neq 0$

**Find:** For given  $x$ , set of actions satisfying exponential stability criterion even in worst case

$$\mathcal{C}(x) \equiv \left\{ u : \left( \sup_{w : \|w\|_2 \leq \|Cx + Du\|_2} \dot{V}(x) \right) \leq -\alpha V(x) \right\}$$

$$\Rightarrow \left\{ u : \|k_1(x) + Du\|_2 \leq k_2(x) + k_3(x)^T u \right\}$$

*Convex (non-empty) set in  $u(t)$*

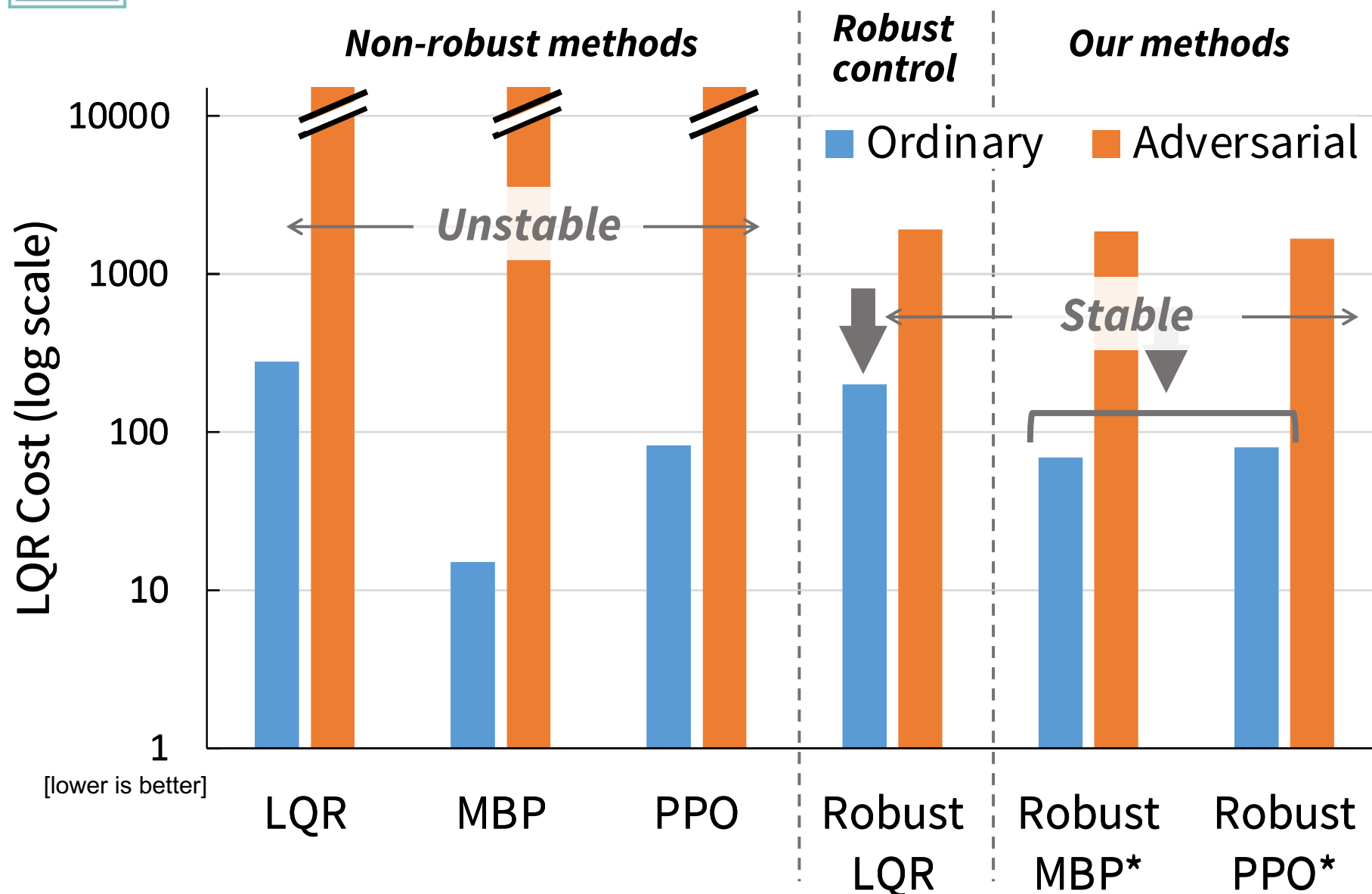


Note:  $t$ -dependence has been dropped for brevity





# Illustrative results: Synthetic NLDI system



**Improved**  
“average-case”  
performance over  
robust baselines

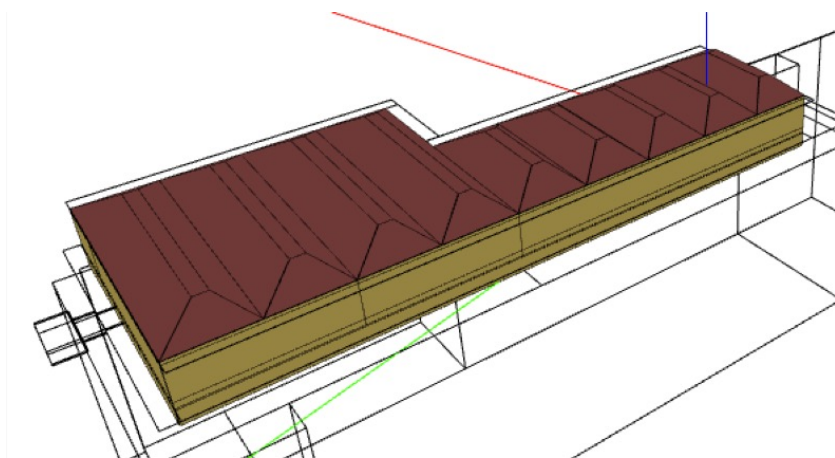
**Provably stable**  
under “worst-case”  
dynamics (unlike  
non-robust baselines)

**Downside:** Speed /  
computational cost

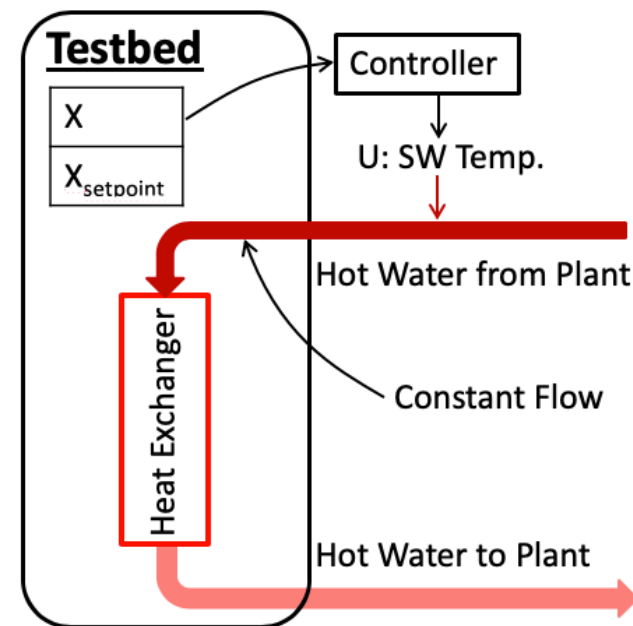


# Energy-efficient heating and cooling

**Goal:** Control the HVAC supply water temperature to minimize energy use, while respecting equipment constraints and maintaining thermal comfort



**Intelligent Workplace**  
**Margaret Morrison Hall, 4<sup>th</sup> Floor**  
 (✿ Zhang & Lam, 2018)

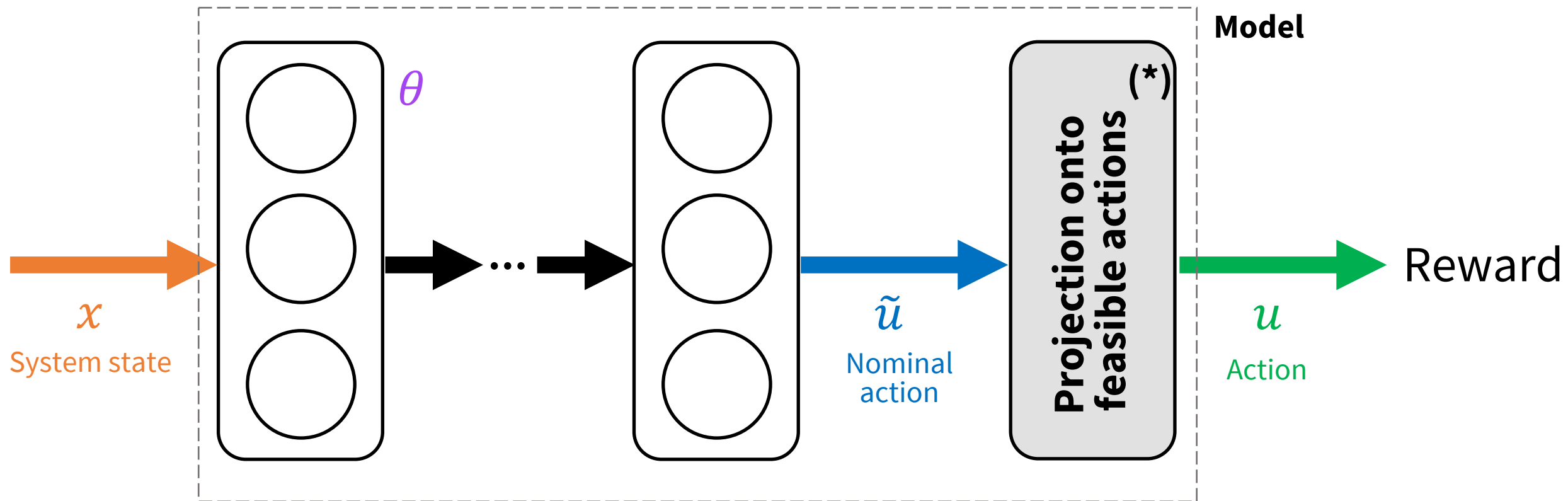


**HVAC Schematic**

Bingqing Chen\*, **Priya L. Donti\***, Kyri Baker, J. Zico Kolter, and Mario Berges. "Enforcing Policy Feasibility Constraints through Differentiable Projection for Energy Optimization." *ACM International Conference on Future Energy Systems (ACM e-Energy) 2021*.



# Differentiable projection onto feasible actions





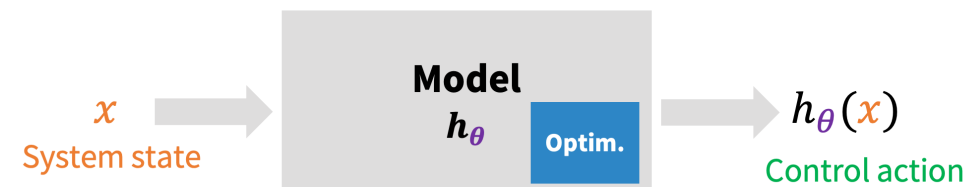
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### Settings:

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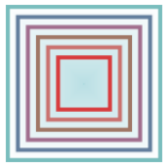


**Insight:** Project outputs of neural network onto a set of “safe” actions

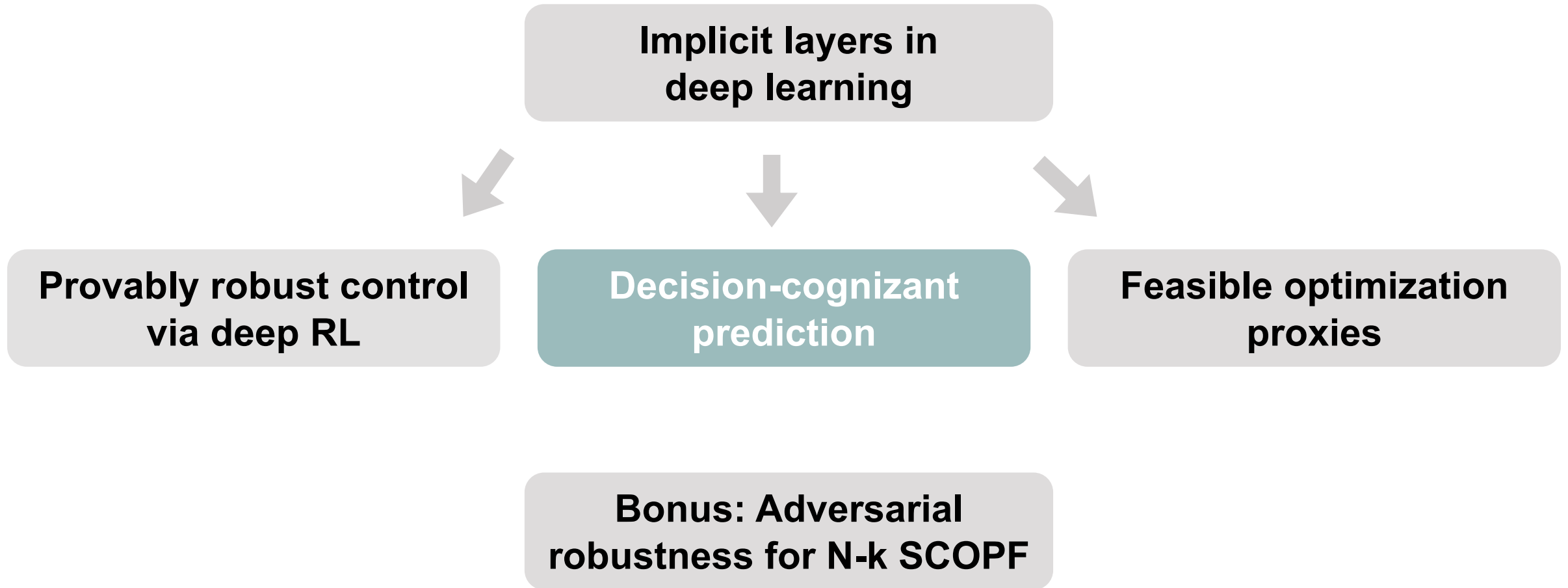
- Obtain safe actions using domain knowledge
- Differentiable projection (implicit layer) = end-to-end training

### Future directions:

- Leveraging more modern control theoretic formulations
- Improving computational costs / scaling to larger systems



# Talk outline

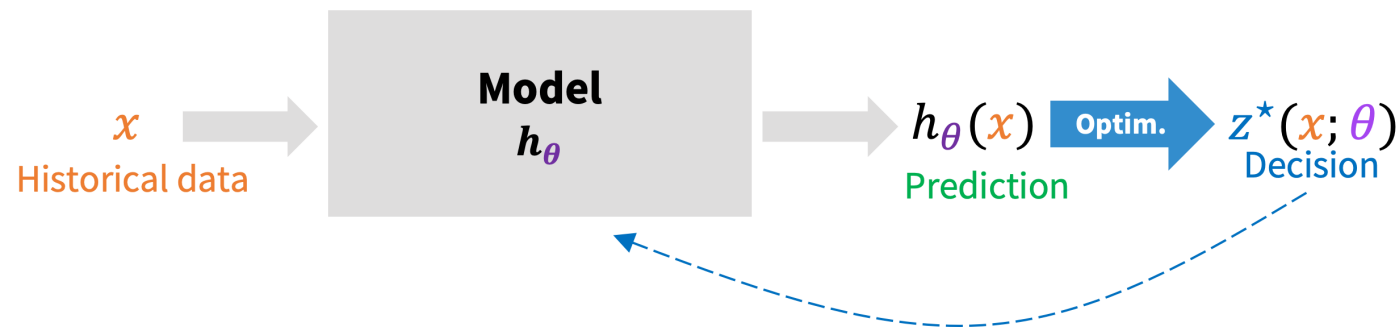




# Overview: Decision-cognizant prediction

**Motivation:** Predictive methods operate within some larger decision-making process but do not often take this into account, potentially leading to critical mistakes.

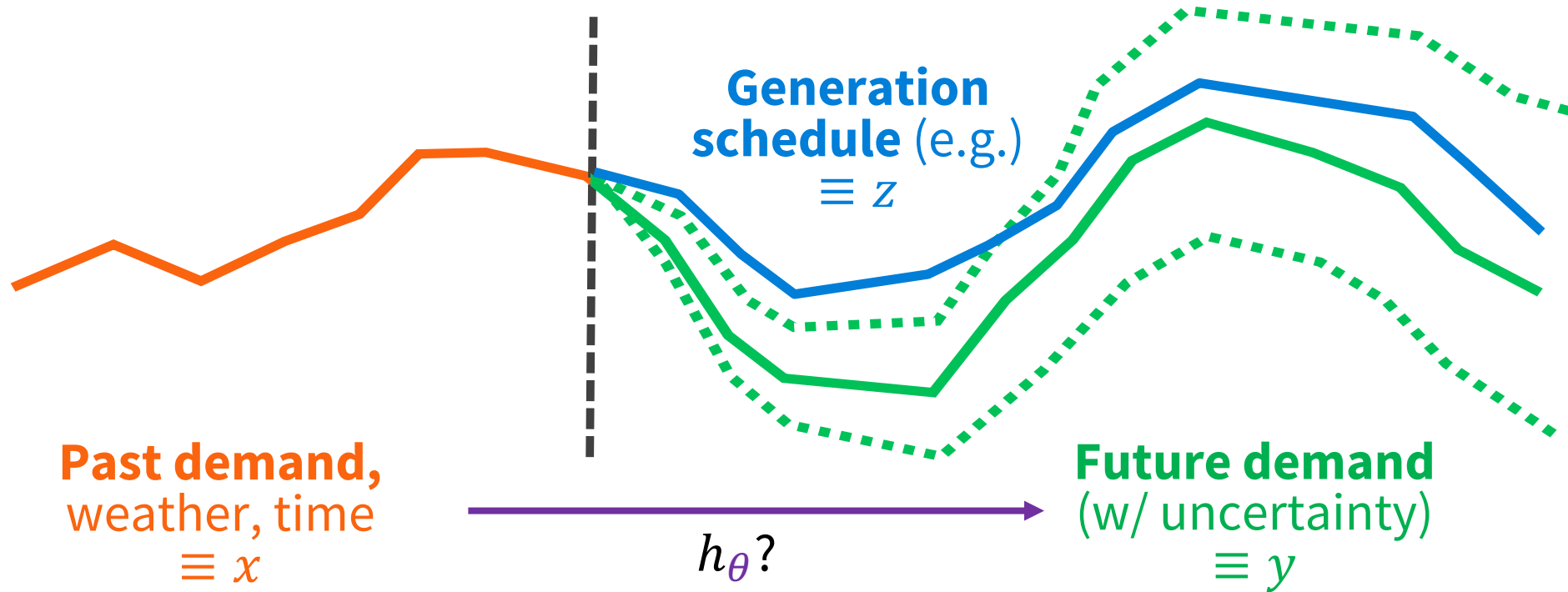
**Approach:** Construct decision-cognizant model using implicit function(s) in objective



**Setting:** Decision-cognizant electricity demand forecasting



# Decision-cognizant demand forecasting



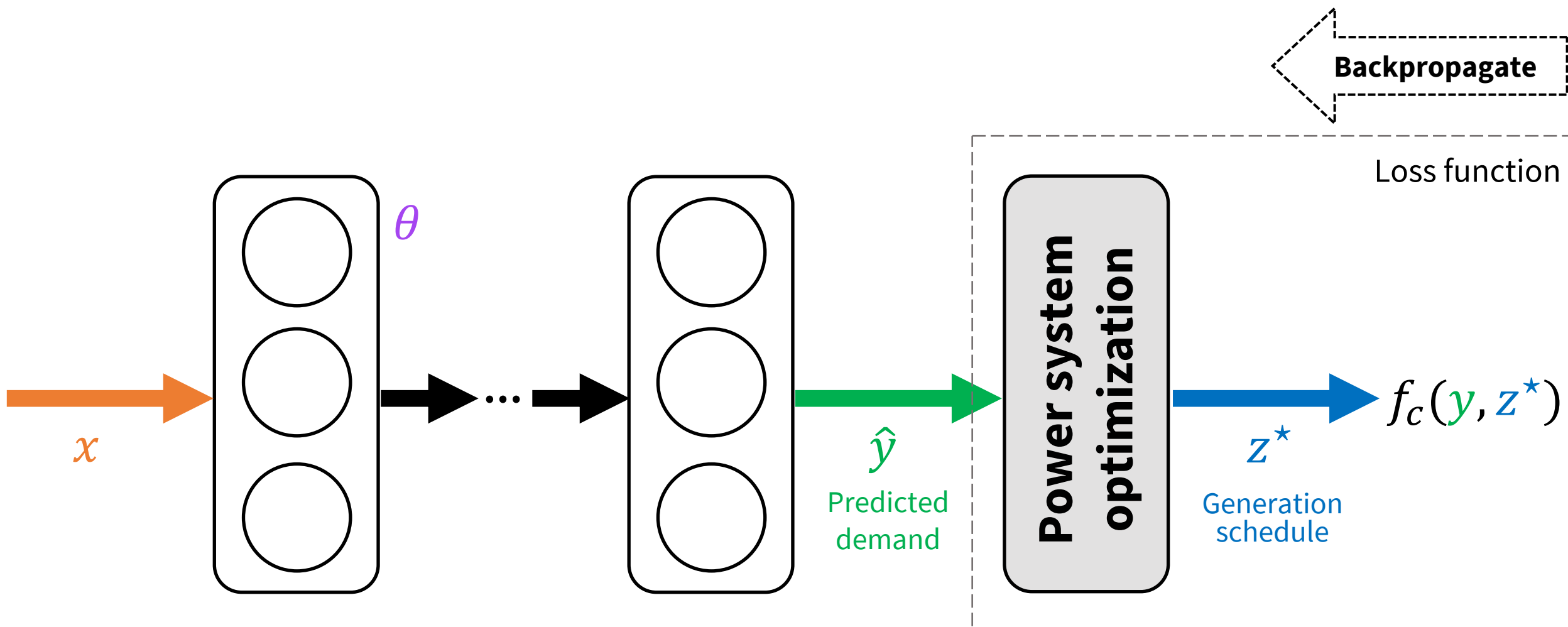
**Goal:** Optimize for quality of generation schedule when we observe actual demands

$$\underset{\theta}{\text{minimize}} f_c(y, z^*(x; \theta))$$

Priya L. Donti, Brandon Amos, and J. Zico Kolter. "Task-based end-to-end model learning in stochastic optimization."  
Conference on Neural Information Processing Systems (NeurIPS) 2017.

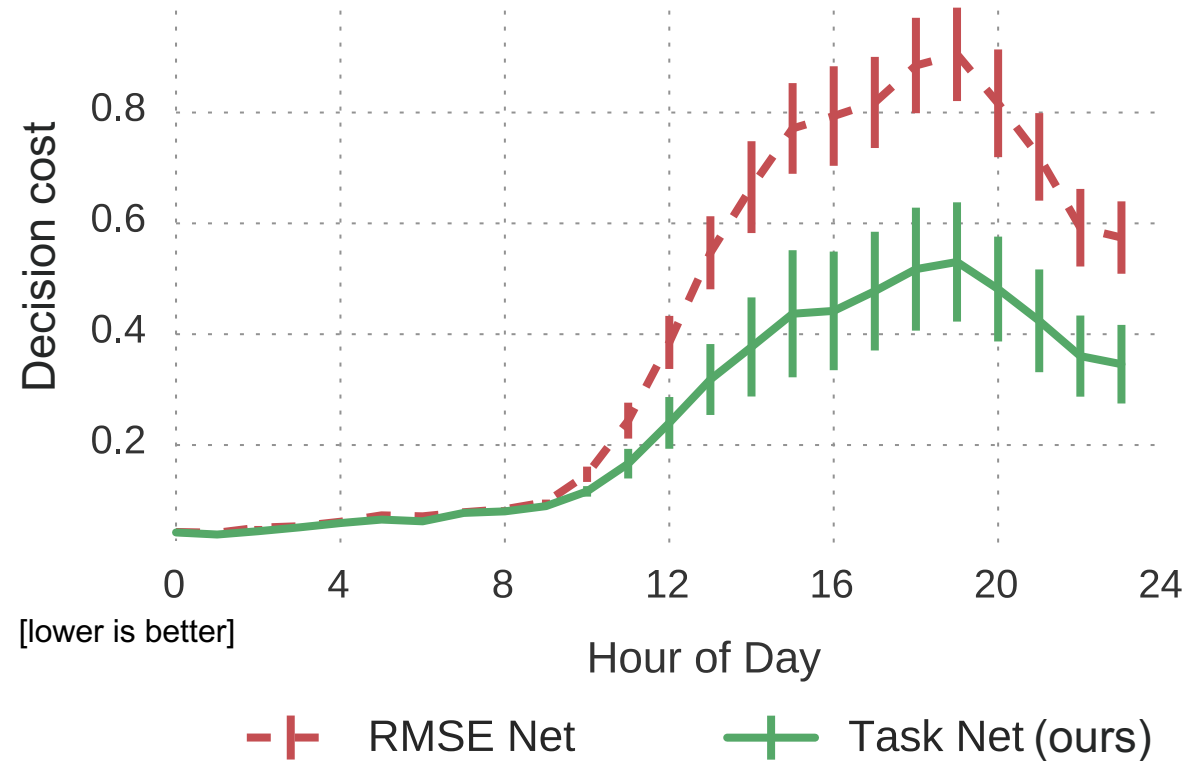
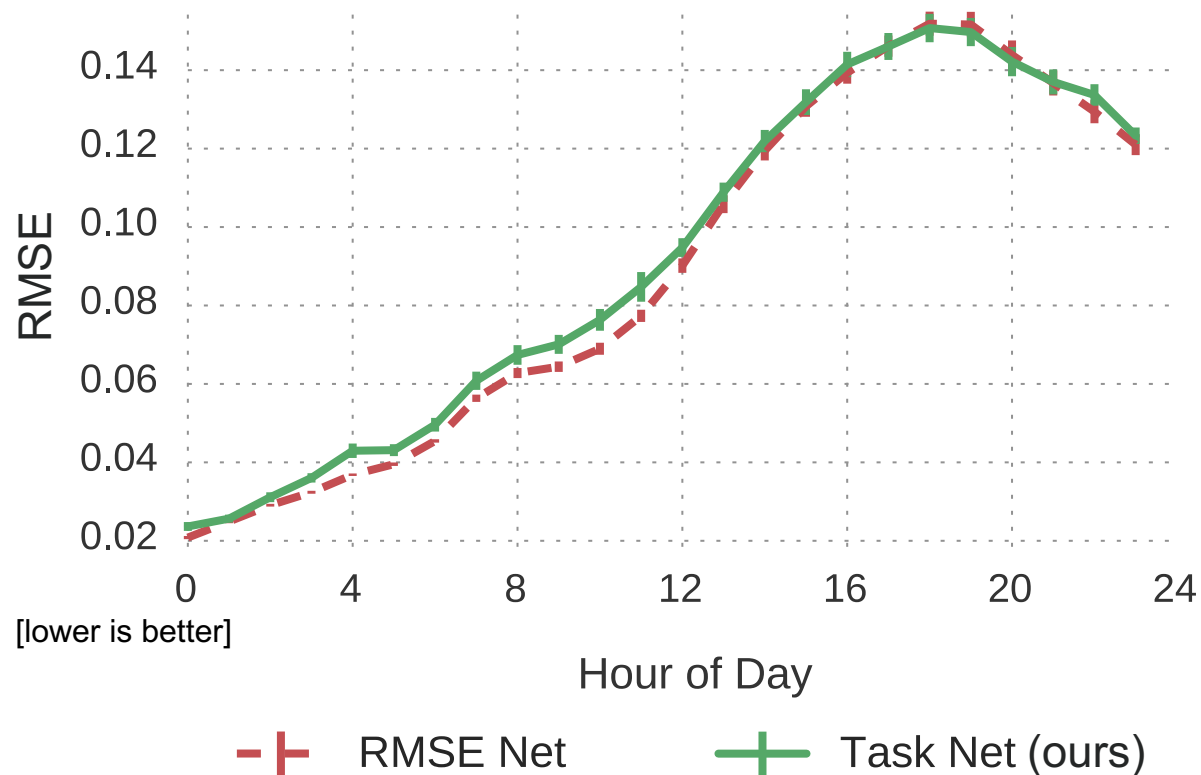


# Decision-cognizant model





# Decision-cognizant approach can dramatically improve generation scheduling outcomes



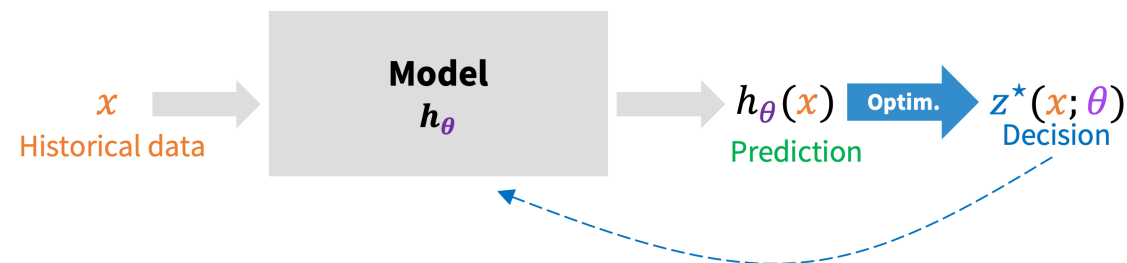
Decision-cognizant approach gives ~**39%** improvement in decision cost.



## Summary: Decision-cognizant prediction

**Motivation:** Predictive methods operate within some larger decision-making process but do not often take this into account, potentially leading to critical mistakes.

**Setting:** Electricity demand forecasting



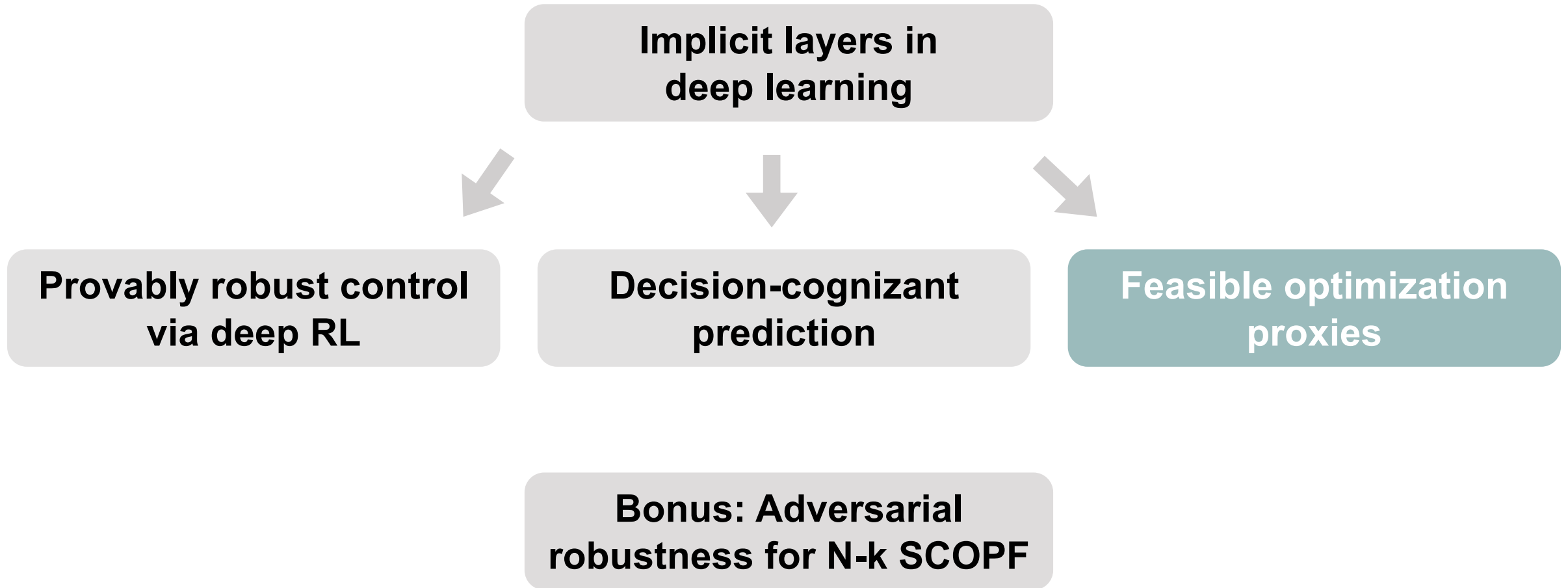
**Insight:** Incorporate knowledge of downstream decision-making process into the loss function, using implicit layers (differentiable optimization).

**Future directions:**

- Incorporating larger / more realistic decision-making procedures
- Extension to additional settings (e.g., end-to-end modeling + control)
- Understanding tradeoffs between decision-cognizant vs. decision-agnostic models



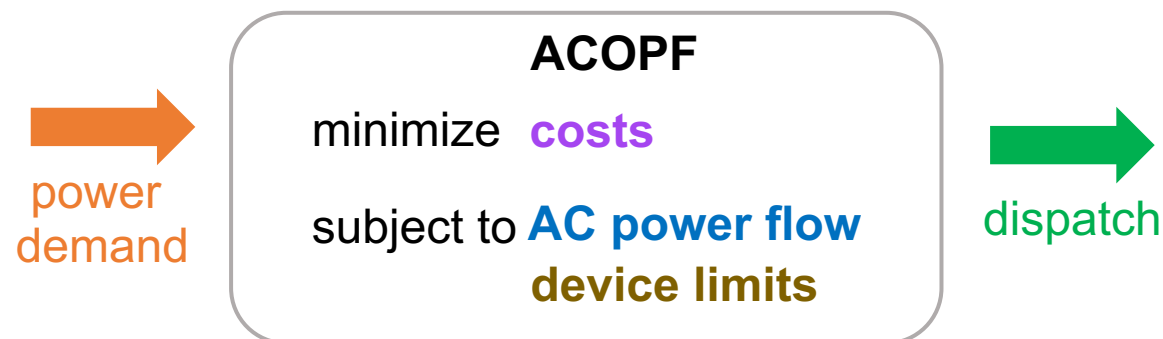
# Talk outline



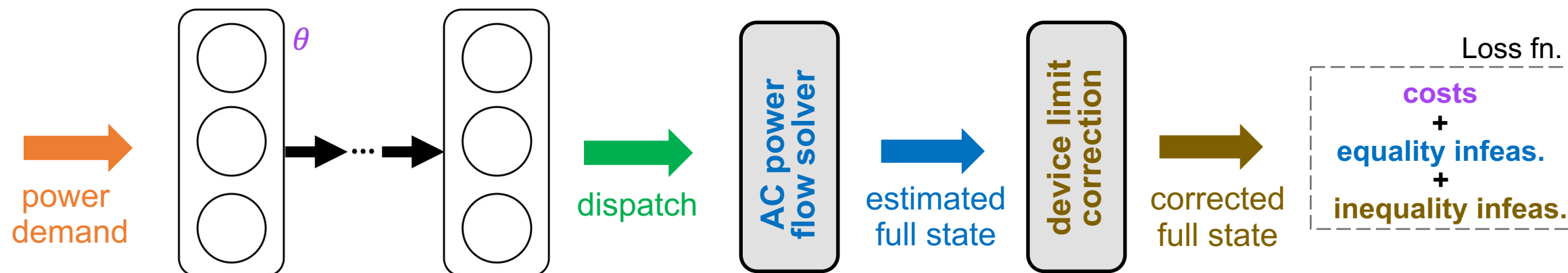


# Feasible optimization proxies

**Goal:** Provide fast, feasible approx. to AC optimal power flow (ACOPF)



**Approach:**



**Note:** Learns directly from problem specification (no training labels)

Priya L. Donti\*, David Rolnick\*, and J. Zico Kolter. "DC3: A learning method for optimization with hard constraints."  
*International Conference on Learning Representations (ICLR) 2021.*



# Approximating ACOPF: 57-bus test case

Comparable  
objective value

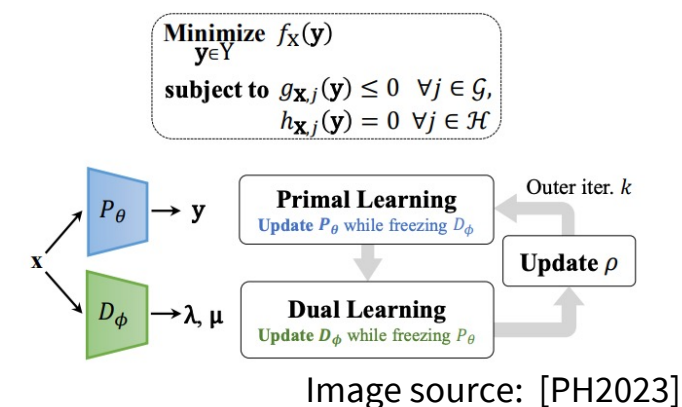
Satisfies all constraints  
(unlike baselines)

10x faster  
than IPOPT

	Objective value	Max equality violation	Mean equality violation	Time (s)
IPOPT	$3.81 \pm 0.00$	$0.00 \pm 0.00$	$0.00 \pm 0.00$	$0.949 \pm 0.002$
Baseline NN	—	$0.19 \pm 0.01$	$0.03 \pm 0.00$	—
<b>Our approach</b>	$3.82 \pm 0.00$	$0.00 \pm 0.00$	$0.00 \pm 0.00$	$0.089 \pm 0.000$

## Future directions:

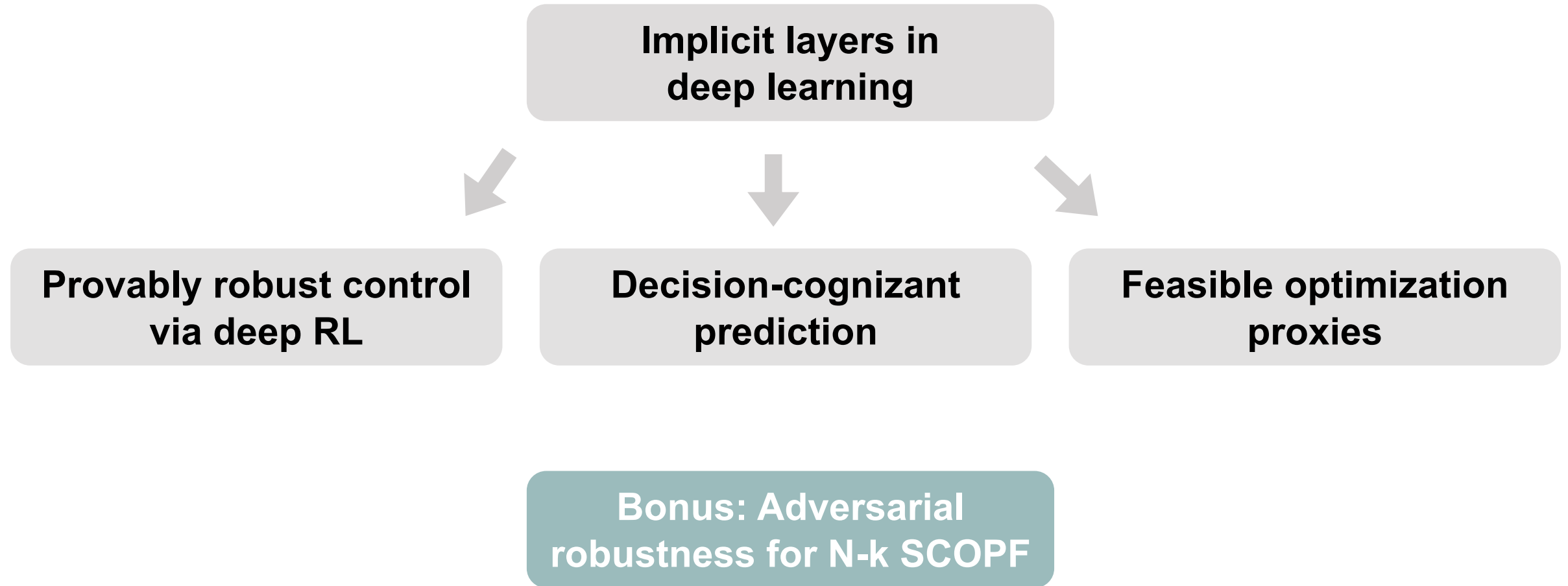
- Larger scale trials (fitting on a GPU)
- Mixed-integer problems (e.g., unit commitment)
- Generalization over topologies (e.g., via GNNs)
- Combinations with frameworks like PDL [PH2023]



[PH2023] Seonho Park, Pascal Van Hentenryck. "Self-Supervised Primal-Dual Learning for Constrained Optimization." *AAAI* (2023).

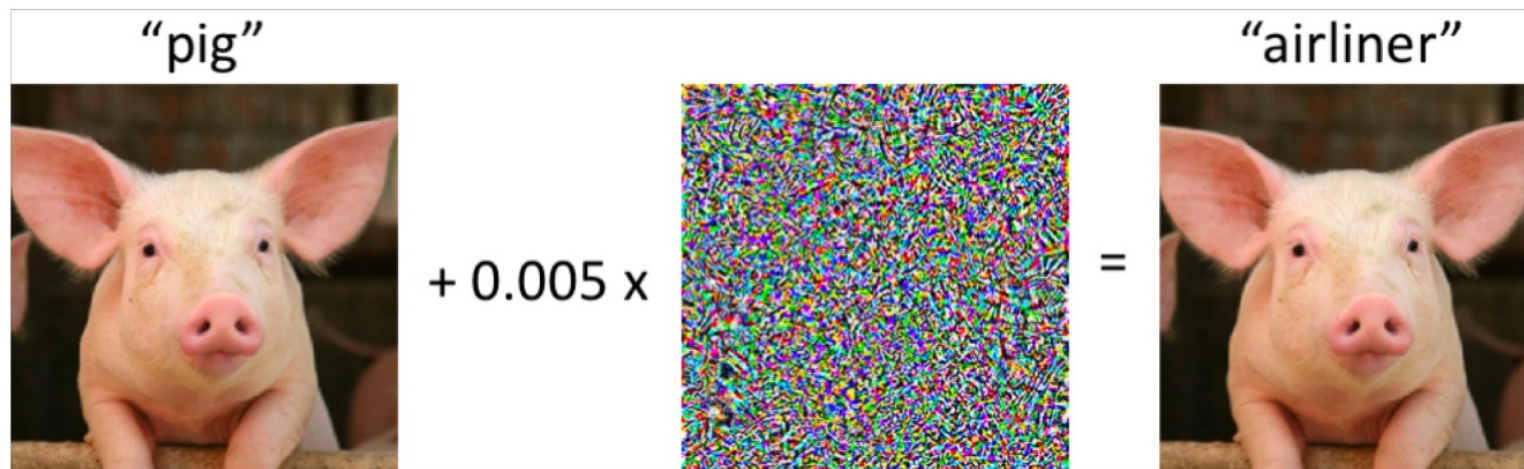


# Talk outline





# Adversarially robust deep learning



Part II: Training a robust classifier

$$\min_{\theta} \sum_{x, y \in S} \max_{\delta \in \Delta} \text{Loss}(x + \delta, y; \theta)$$

Part I: Creating an adversarial example (or ensuring one does not exist)



## Drawing inspiration from adversarially robust DL

**Adversarially robust deep learning:** Pick **neural network parameters** to bound the cost of any **worst-case perturbation**

- Required scalable gradient-based optimization methods

**Security-constrained OPF:** Pick **dispatch** to bound the cost of **worst-case contingencies**

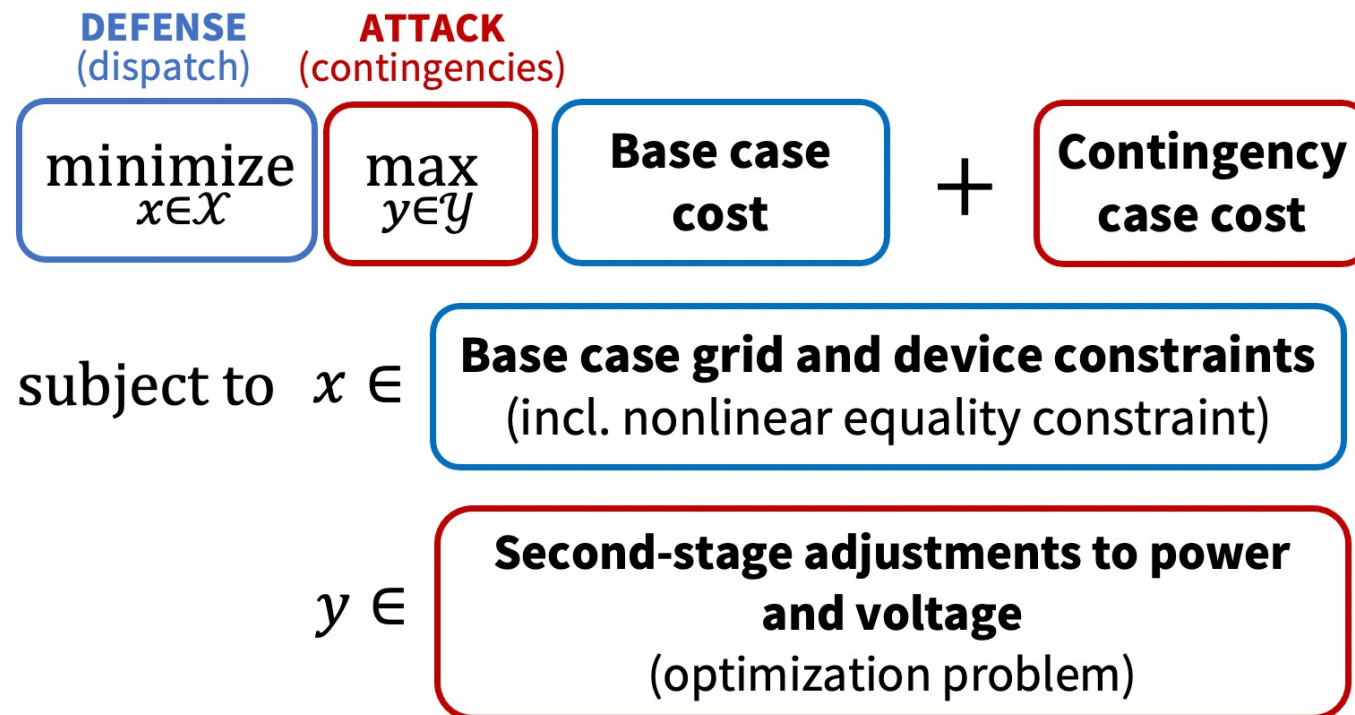
- Leverage similar scalable gradient-based optimization methods?





# Adversarial robustness for N-k SCOPF

**N-k security-constrained optimal power flow (SCOPF):** Schedule power to be robust to potentially  $k$  simultaneous generator or line failures (contingencies)



**Step 0:** Formulate as bilevel (attacker-defender) opt. over dispatch and continuous outer relaxation of contingencies

**Step 1 (“attack stage”):** Find worst-case contingency via a few steps of projected gradient ascent (with implicit diff.)

**Step 2 (“defense stage”):** Update dispatch to improve robustness against worst case contingency (e.g., via projected gradient descent or efficient Gauss-Seidel approach)

Priya L. Donti\*, Aayushya Agarwal\*, Neeraj Vijay Bedmutha, Larry Pileggi, and J. Zico Kolter. "Adversarially Robust Learning for Security-Constrained Optimal Power Flow." *Conference on Neural Information Processing Systems (NeurIPS) 2021*.



## Illustrative results (4622-bus system)

**3-4x improvement** over OPF for N-2/N-3 SCOPF, in **only 21 minutes on a laptop**

Contingency type	N-1	N-2	N-3
Scenarios tested	6,133	359,712	428,730
OPF violations	59	10,572	4,086
CAN $\partial$ Y violations*	36	3,580	1,122

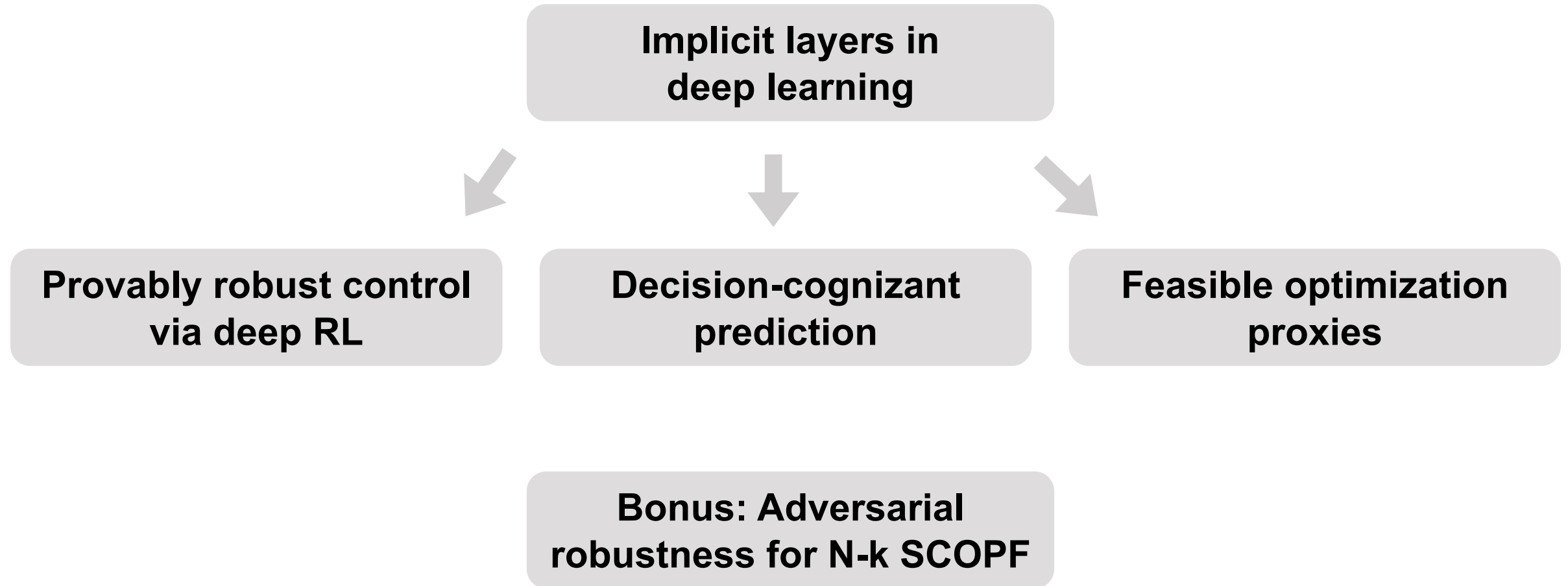
\* our approach

**Note:** Comparable N-1 SCOPF performance, and superior N-2 and N-3 performance, to ARPA-E GO Competition baselines

**See also:** Results on stochastic OPF for 11,615-bus system (PSCC 2022)



# Talk outline





# Enablers for next-gen. optimization & control

**More openness in data**, beyond only bilateral agreements and limited access

- Can include sharing of synthetic data

**Simulators and test beds**, with realistic/diverse scenarios and easy-to-use interfaces

- Includes digital twins, but also simpler frameworks (e.g., Grid2Op)
- Need for *progression pathways* from basic to advanced simulators/test beds

**Evaluation metrics / benchmarks:** What does it mean for a method to succeed (or fail)?

**Open-source software**, enabling integration and evaluation of new methods

**Internal research capacity with external exchange:** Enables translation of ideas without sharing difficult-to-share information across organizational boundaries

*Note: None of these enablers are machine learning-specific!*

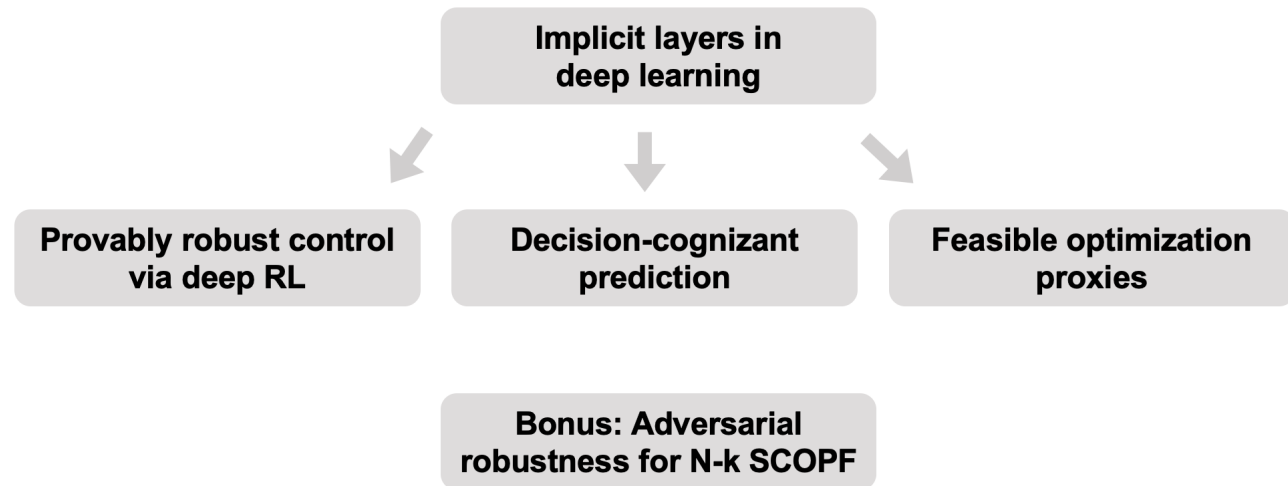


## Closing thoughts

**Implicit layers in deep learning:**  
powerful paradigm for bridging ML  
with power systems specifications

Going from theory to practice  
requires deep interdisciplinary  
**collaboration** and research-to-  
deployment **infrastructure**

Reach out if you'd like to chat, and  
check out the Climate Change AI  
network ([www.climatechange.ai](http://www.climatechange.ai))



**Priya L. Donti:** donti@mit.edu



# Backup slides



# ML for power systems: Recurring themes

See also: Donti, P. L. & Kolter, J. Z. (2021). Machine learning for sustainable energy systems. *Annual Review of Environment and Resources*, 46, 719-747.



# ML for power systems: Recurring themes

**Distilling raw data into insights** (GHG emissions, solar panels, vegetation)

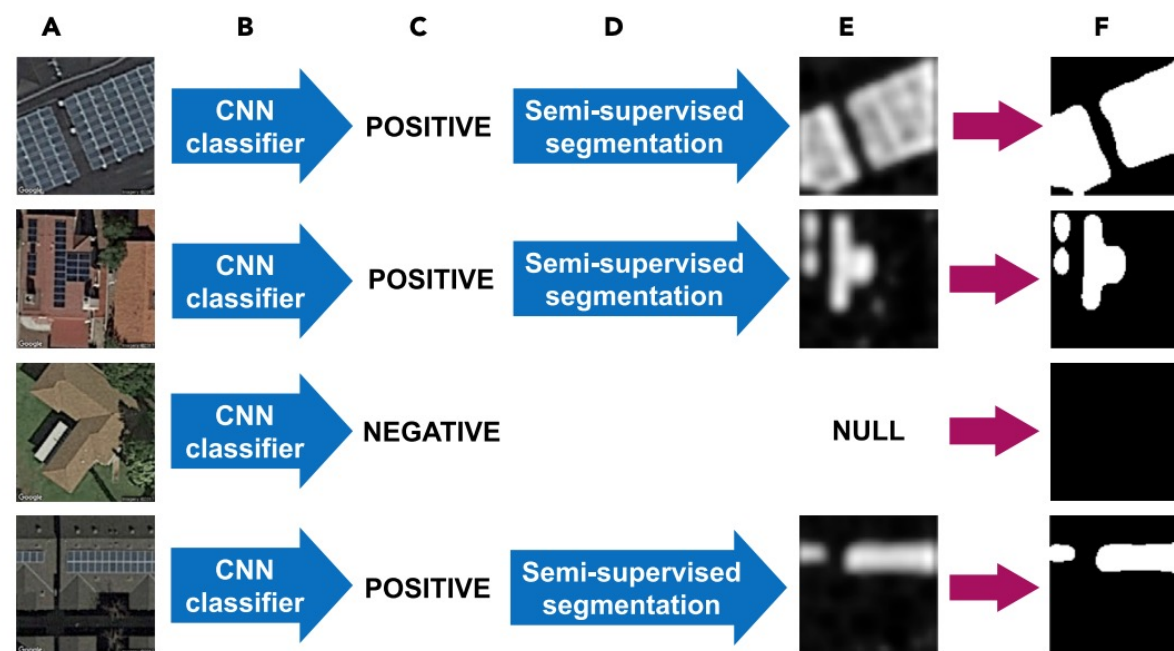


Image source: Yu, Wang, Majumdar, Rajagopal (2018)

See also: Donti, P. L. & Kolter, J. Z. (2021). Machine learning for sustainable energy systems. *Annual Review of Environment and Resources*, 46, 719-747.





# ML for power systems: Recurring themes

**Distilling raw data into insights** (GHG emissions, solar panels, vegetation)

**Forecasting** (renewable energy, marginal/average emissions, prices)

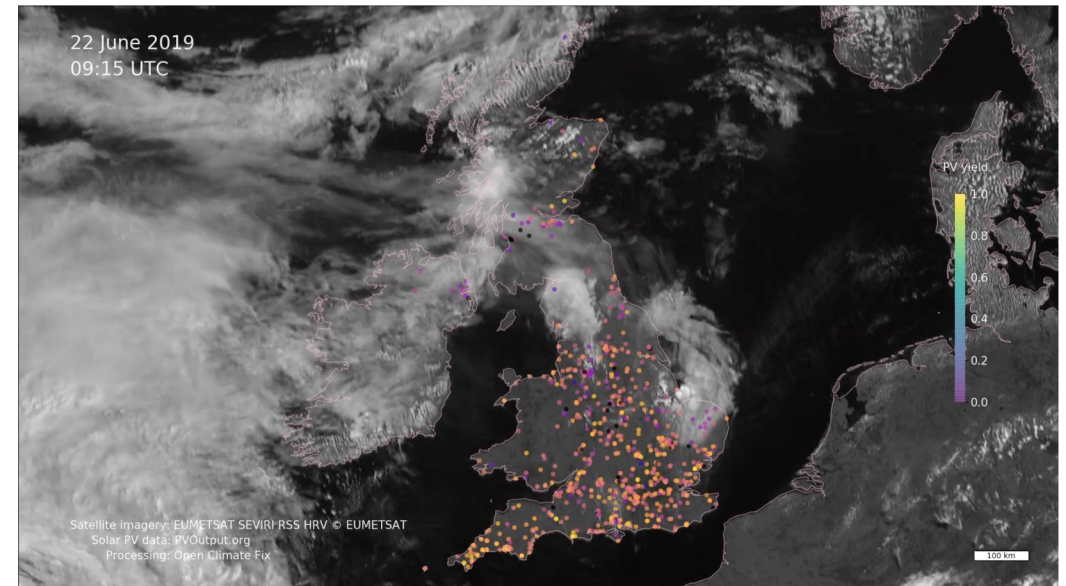


Image source: Open Climate Fix

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**Distilling raw data into insights** (GHG emissions, solar panels, vegetation)

**Forecasting** (renewable energy, marginal/average emissions, prices)

**Fast and dynamic optimization** (power scheduling, MPPT)



Image source: L2RPN Challenge

See also: Donti, P. L. & Kolter, J. Z. (2021). Machine learning for sustainable energy systems. *Annual Review of Environment and Resources*, 46, 719-747.



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**Distilling raw data into insights** (GHG emissions, solar panels, vegetation)

**Forecasting** (renewable energy, marginal/average emissions, prices)

**Fast and dynamic optimization** (power scheduling, MPPT)

**Predictive maintenance**

(resilient infrastructure, methane leaks)



Image source: [EPRI Journal \(2019\)](#)

See also: Donti, P. L. & Kolter, J. Z. (2021). Machine learning for sustainable energy systems. *Annual Review of Environment and Resources*, 46, 719-747.



# ML for power systems: Recurring themes

**Distilling raw data into insights** (GHG emissions, solar panels, vegetation)

**Forecasting** (renewable energy, marginal/average emissions, prices)

**Fast and dynamic optimization** (power scheduling, MPPT)

**Predictive maintenance**

(resilient infrastructure, methane leaks)

**Accelerated science**

(batteries, solar, electrofuels, fusion)

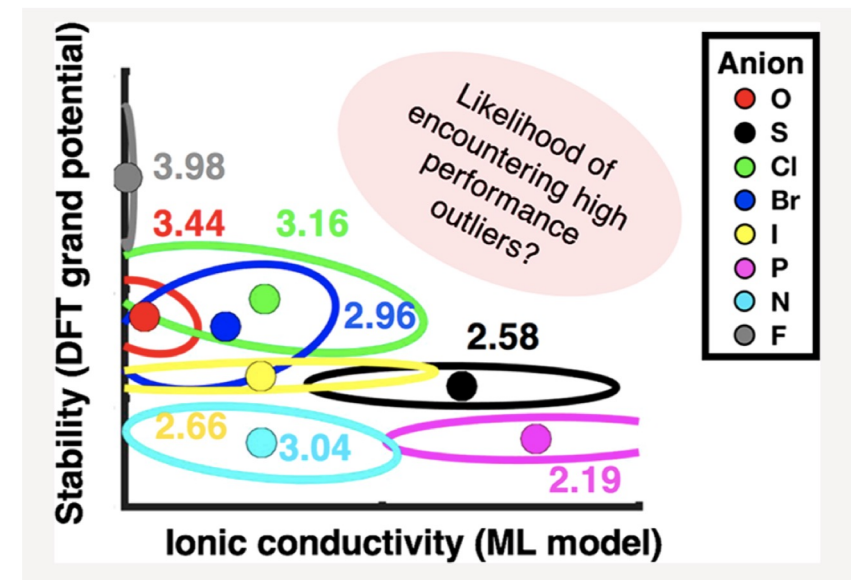


Image source: Sendek et al. (2020)

See also: Donti, P. L. & Kolter, J. Z. (2021). Machine learning for sustainable energy systems. *Annual Review of Environment and Resources*, 46, 719-747.



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**Data management**

(data matching/fusion, data generation)

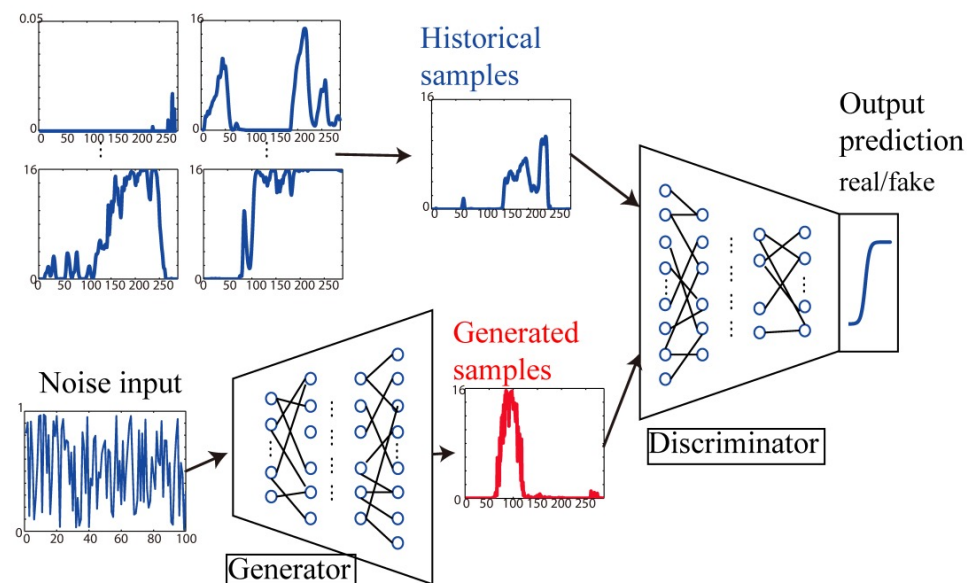
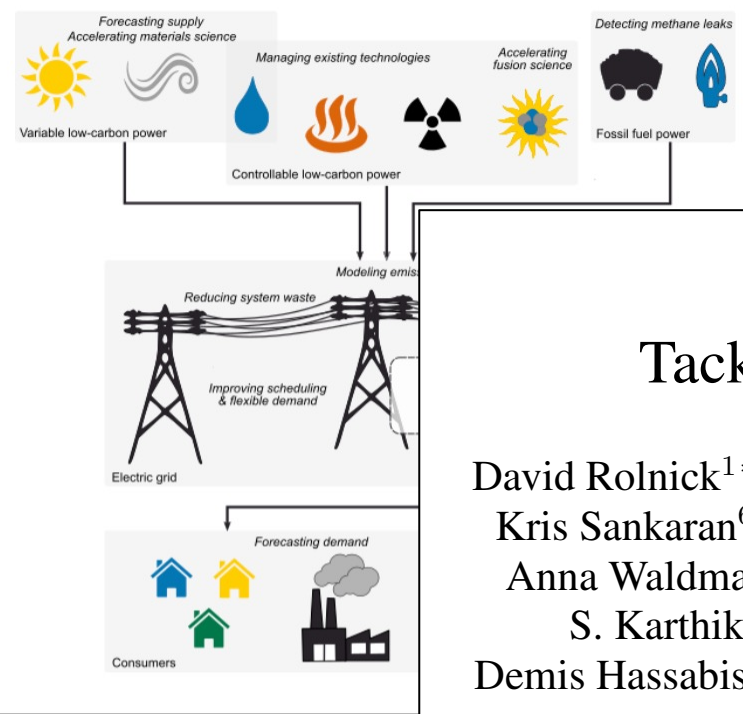


Image source: Chen, Wang, Kirschen, Zhang (2018)

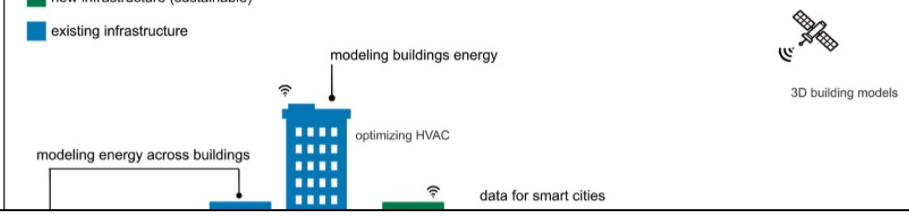
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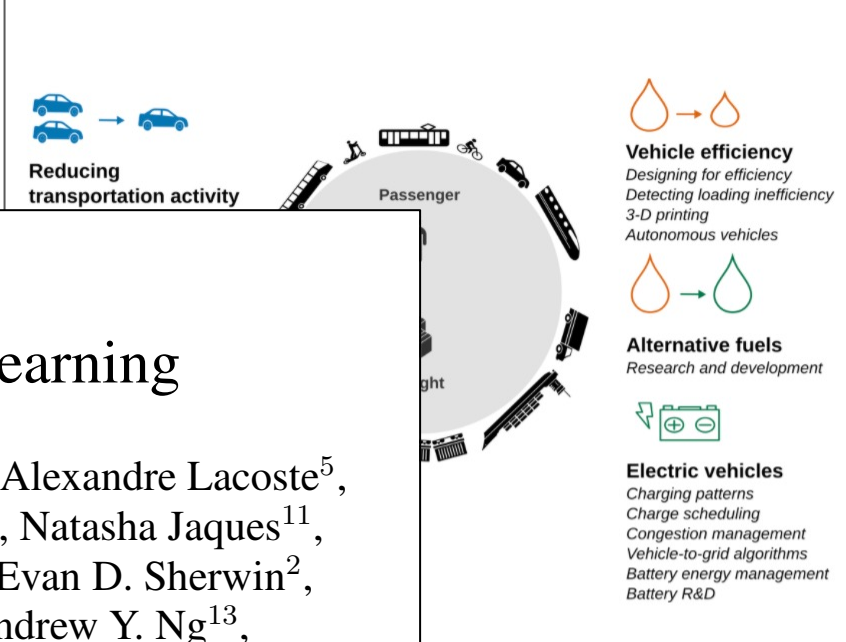
# Electricity systems



# Buildings & cities



# Transportation

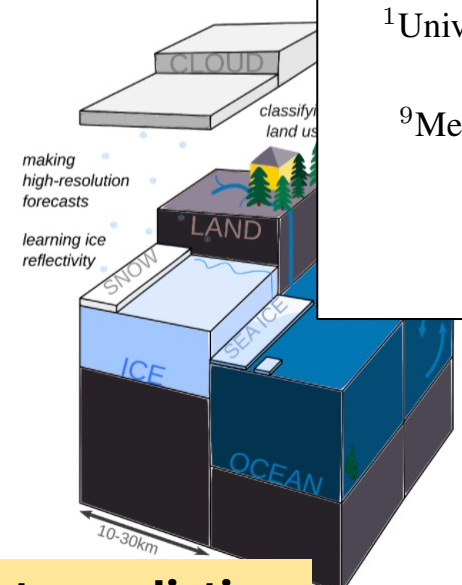


# Tackling Climate Change with Machine Learning

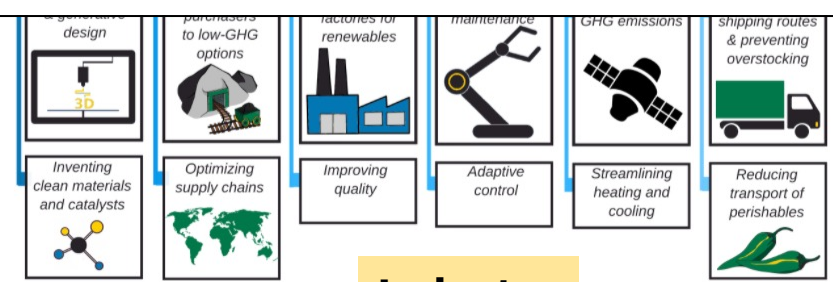
David Rolnick<sup>1\*</sup>, Priya L. Donti<sup>2</sup>, Lynn H. Kaack<sup>3</sup>, Kelly Kochanski<sup>4</sup>, Alexandre Lacoste<sup>5</sup>, Kris Sankaran<sup>6,7</sup>, Andrew Slavin Ross<sup>8</sup>, Nikola Milojevic-Dupont<sup>9,10</sup>, Natasha Jaques<sup>11</sup>, Anna Waldman-Brown<sup>11</sup>, Alexandra Luccioni<sup>6,7</sup>, Tegan Maharaj<sup>6,7</sup>, Evan D. Sherwin<sup>2</sup>, S. Karthik Mukkavilli<sup>6,7</sup>, Konrad P. Kording<sup>1</sup>, Carla Gomes<sup>12</sup>, Andrew Y. Ng<sup>13</sup>, Demis Hassabis<sup>14</sup>, John C. Platt<sup>15</sup>, Felix Creutzig<sup>9,10</sup>, Jennifer Chayes<sup>16</sup>, Yoshua Bengio<sup>6,7</sup>

<sup>1</sup>University of Pennsylvania, <sup>2</sup>Carnegie Mellon University, <sup>3</sup>ETH Zürich, <sup>4</sup>University of Colorado Boulder, <sup>5</sup>Element AI, <sup>6</sup>Mila, <sup>7</sup>Université de Montréal, <sup>8</sup>Harvard University, <sup>9</sup>Mercator Research Institute on Global Commons and Climate Change, <sup>10</sup>Technische Universität Berlin, <sup>11</sup>Massachusetts Institute of Technology, <sup>12</sup>Cornell University, <sup>13</sup>Stanford University, <sup>14</sup>DeepMind, <sup>15</sup>Google AI, <sup>16</sup>Microsoft Research

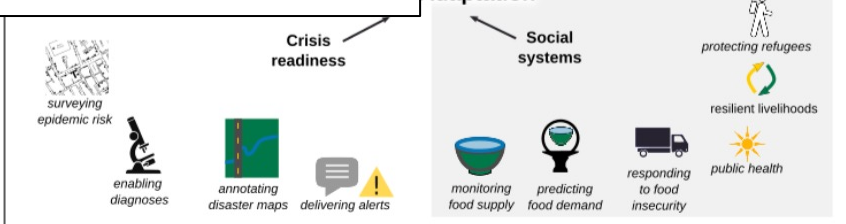
# Climate prediction



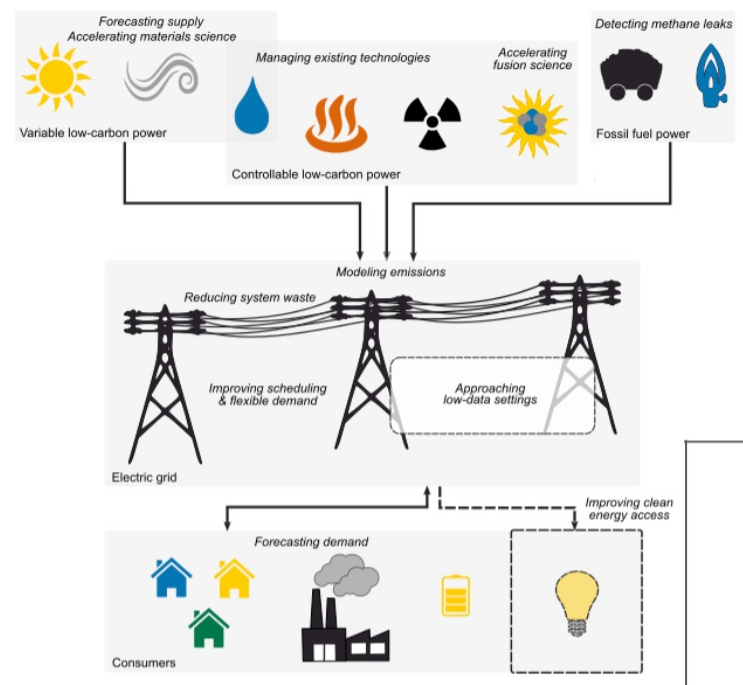
# Industry



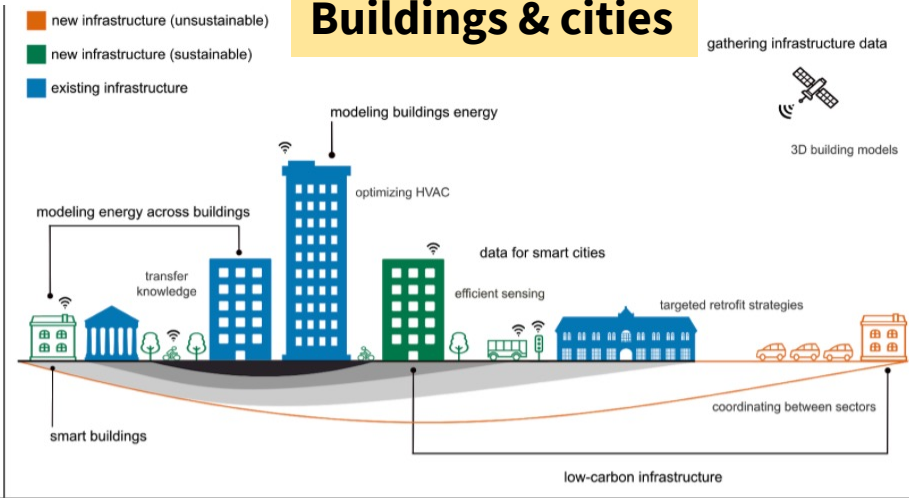
# Societal adaptation



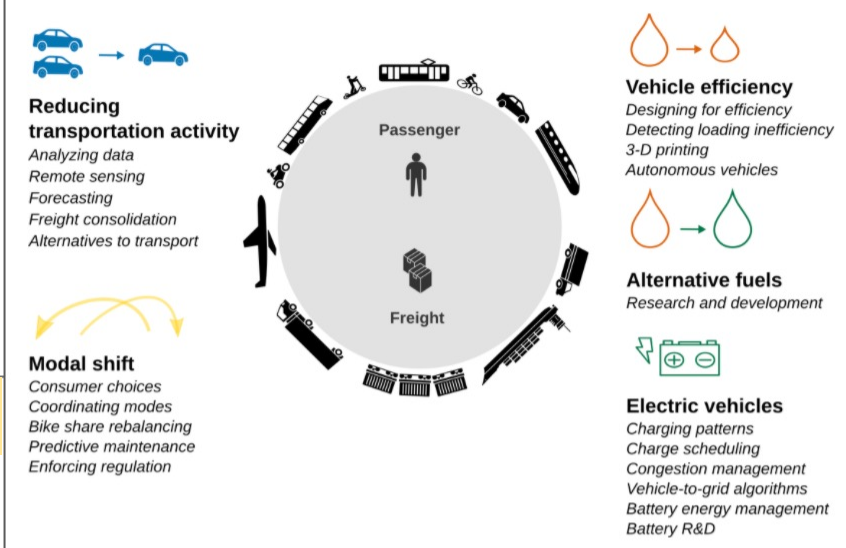
# Electricity systems



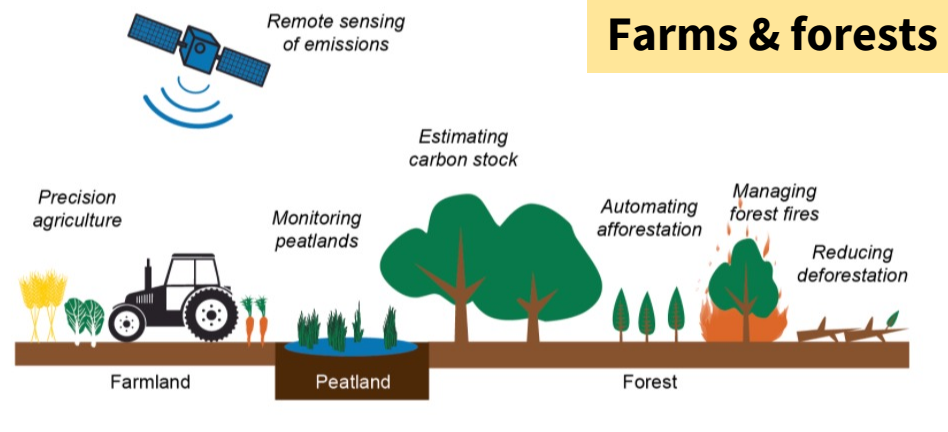
# Buildings & cities



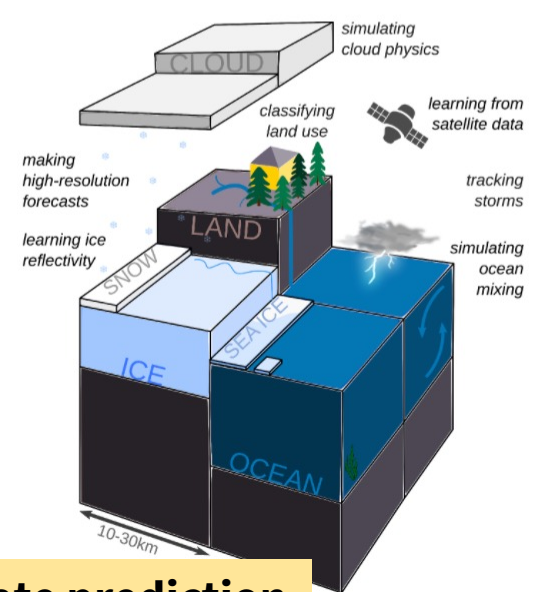
# Transportation



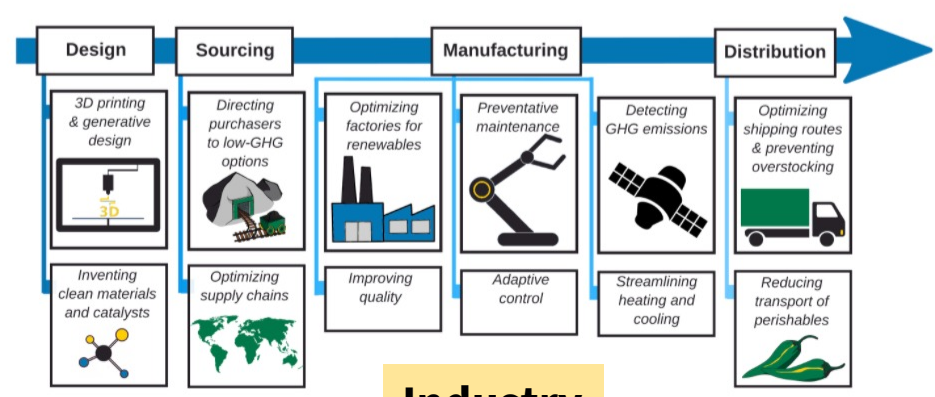
# Farms & forests



# Climate prediction



# Industry



# Societal adaptation

