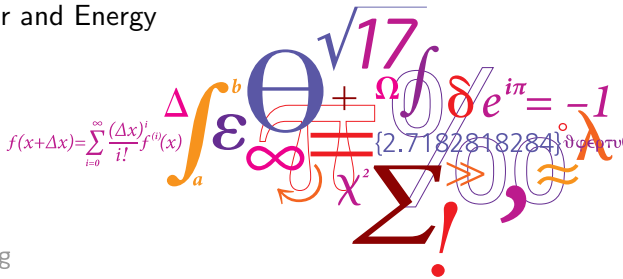
Semidefinite Programming for Power System Stability and Optimization

1st Hellenic International Power Group Workshop, NTUA, Greece

Spyros Chatzivasileiadis

DTU Center for Electric Power and Energy



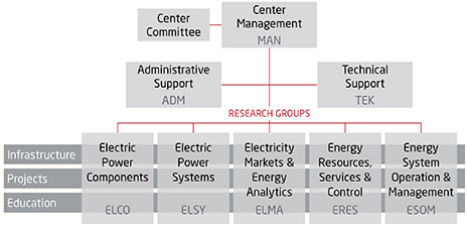
DTU Electrical Engineering
Department of Electrical Engineering

Outline

- Who are we? ... and what we do
- What is semidefinite programming (SDP) ?
- Power System Stability Assessment and SDP
- Power System Optimization and SDP

DTU Center for Electric Power and Energy

- **Established 15 Aug. 2012; merger of existing units (Lyngby+Risø)**
- One of the strongest university centers in Europe with $\sim$ 100 employees



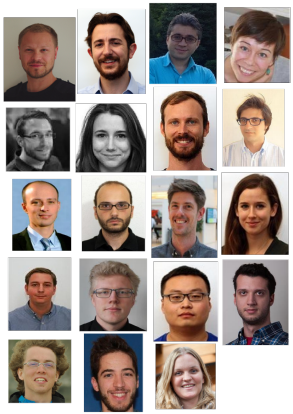
Mission: Provides cutting-edge research, education and innovation in the field of electric power and energy to meet the future needs of society regarding a reliable, cost efficient and environmentally friendly energy system

- **BSc & MSc:** Electrical Engineering, Wind Energy, Sustainable Energy
- **Direct Support from:** Energinet.dk, Siemens, DONG Energy, Danfoss

DTU ranked world 2nd in Energy Science and Engineering(!)

The Energy Analytics & Markets group

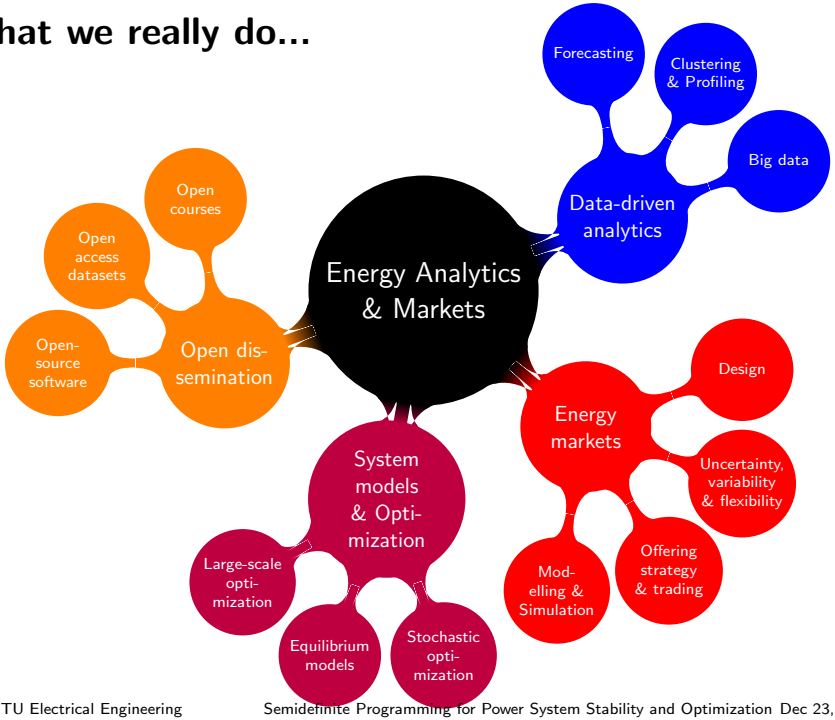
One of the 5 groups of the Center for Electric Power and Energy,
Department of Electrical Engineering



- **Resources:** (10 nationalities)
  - *Faculty:* 1 Prof, 2 Assist. Profs.
  - *Junior:* 3 post-doc fellows, 9 Ph.D. students (+2 externals), 2-3 research assistants
  - + student helpers, and Ph.D. guests from China, Brazil, US, Spain, France, Italy, Netherlands, Germany, etc.
- **Projects** (active in 2016):
  - **EU:** BestPaths
  - **Danish:** 5s, EcoGrid 2.0, CITIES, EnergyLab Nordhavn, EnergyBlock
  - **Danish-Chinese:** PROAIN

- **Education:** Various courses on renewables forecasting, optimization, and electricity markets
- (hopefully) recognized leading expertise in energy analytics and markets

What we really do...



Research Topics (Selection)

- Optimal operation of combined heat, gas, and electricity networks
- Game theoretical approaches for electricity market participants
- Spatiotemporal forecasting for wind, solar, and energy demand
- Stochastic electricity market design and value of information
- HVDC optimization and control under uncertainty

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Τι είναι ημιορισμένος προγραμματισμός;

- Είναι ένα συνολο μεθόδων βελτιστοποίησης
- Ψάχνουμε να βρούμε ένα σύνολο μεταβλητών που:
  - ελαχιστοποιούν μια συνάρτηση, π.χ. ελάχιστο κόστος ενέργειας
  - υπόκεινται σε ορισμένους περιορισμούς, π.χ. η τάση κάθε ζυγού πρέπει να βρίσκεται εντός κάποιων ορίων
- αν όλες οι συναρτήσεις είναι γραμμικές $\Rightarrow$ γραμμικός προγραμματισμός
- αν οι συναρτήσεις είναι μη γραμμικές (π.χ. γινόμενα τάσεων) αλλά μπορούμε να τις αναπαραστήσουμε σε μορφή γραμμικού συνδυασμού πινάκων $\Rightarrow$ ημιορισμένος προγραμματισμός

What is Semidefinite Programming? (SDP)

- SDP is the "generalized" form of an LP (linear program)
- Example of a linear program: Suppose a production manager is responsible for scheduling the monthly production levels x_j of a certain product for a planning horizon of twelve months.

Production cost c_j per month	$\min \sum c_j x_j$
subject to:	
total annual demand D	$x_1 + \ldots + x_{12} = D$
maximum production capacity per month m_j	$0 \leq x_j \leq m_j$

What is Semidefinite Programming? (SDP)

- SDP is the "generalized" form of an LP (linear program)

Linear Programming	Semidefinite Programming
$\min c \cdot x$	$\min C \bullet X := \sum_i \sum_j C_{ij} X_{ij}$
subject to:	subject to:
$a_i \cdot x = b_i, \quad i = 1, \ldots, m$ $x \geq 0, \quad x \in R^n$	$A_i \bullet X = b_i, \quad i = 1, \ldots, m$ $X \succeq 0$

- LP: Optimization variables in the form of a vector x.
- SDP: Optimization variables in the form of a semidefinite *matrix* X.

SDP for Power System Stability and Optimization

SDP for Power System Stability

find a feasible X

subject to:

$$A_i \bullet X \succeq 0$$
$$X \succeq 0$$

SDP for Optimal Power Flow

minimize cost of electricity

$$\min C \bullet X$$

subject to:

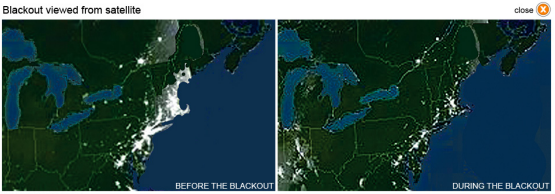
voltage and power flow constraints

$$A_i \bullet X = b_i, \quad i = 1, \ldots, m$$
$$X \succeq 0$$

Robust Power System Stability Assessment with Extensions to Inertia and Topology Control

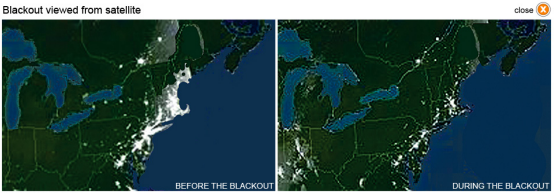
work with:
Thanh Long Vu, Kostya Turitsyn
MIT Mechanical Engineering

Power blackouts



SOURCE: NOAA.gov

Power blackouts



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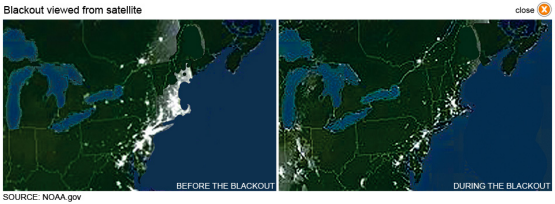
Statistics:

- 2003 US: 55M people; 2011 India: 700M people
- Frequency: $\approx 1hr/year \implies$ economic damage: $\approx 100B\$/year$
- Total electric energy cost in US: $\approx 400B\$/year$

Power blackouts



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Challenges and opportunities:

- New algorithms for better decision-making

Dynamic Security Assessment

- Security = ability to withstand disturbances
- Security Assessment:
  - Screen contingency list every 15 mins
  - Prepare contingency plans for critical scenarios.

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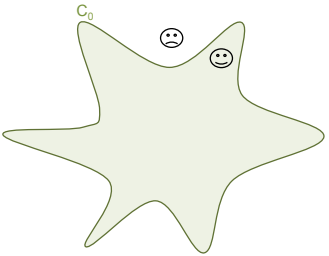
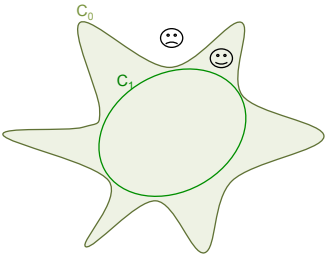
- Dynamic simulations are hard:
  - DAE system with about 10k degrees of freedom
  - Faster than real-time simulations is an open research topic
- Alternative: Energy methods = Security certificates

Security certificates



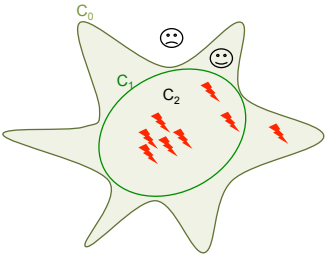
- Security region: non-convex, NP-hard characterization

Security certificates



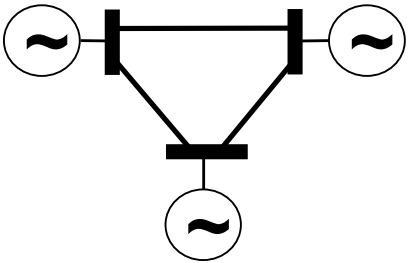
- Security region: non-convex, NP-hard characterization
- Security certificates: tractable sufficient conditions

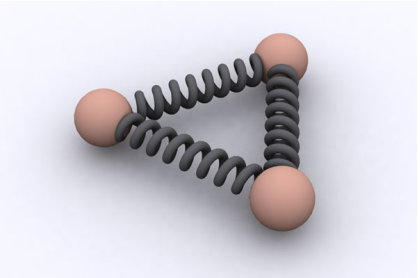
Security certificates



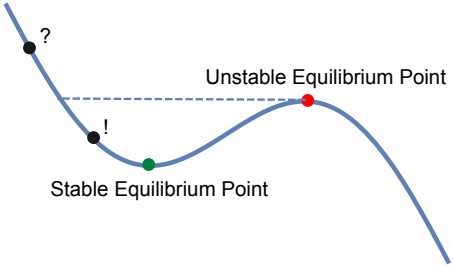
- Security region: non-convex, NP-hard characterization
- Security certificates: tractable sufficient conditions
- Strategy: certify security of most of scenarios with conservative conditions, use simulations for few really dangerous scenarios

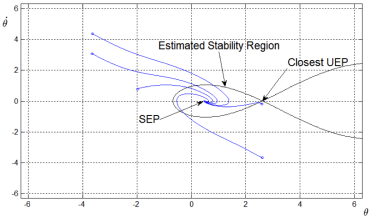
Closest mechanical equivalent to a power system is a mass-spring system





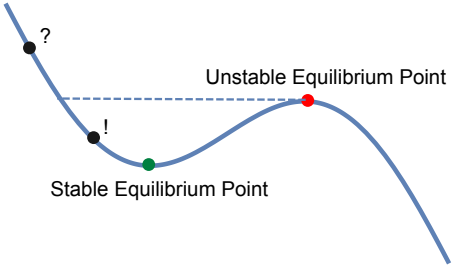
Energy method

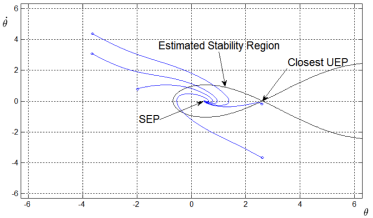




- If $E(\delta_0, \dot{\delta}_0) < E_{CUEP}$, then stable

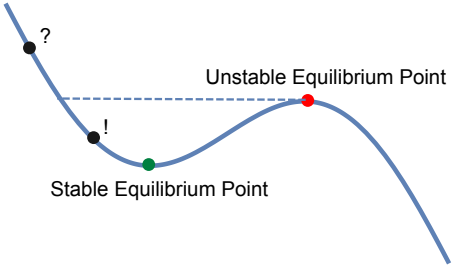
Energy method

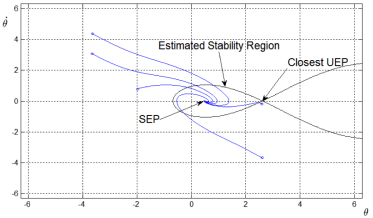




- If $E(\delta_0, \dot{\delta}_0) < E_{CUEP}$, then stable
- Fast transient stability certificate

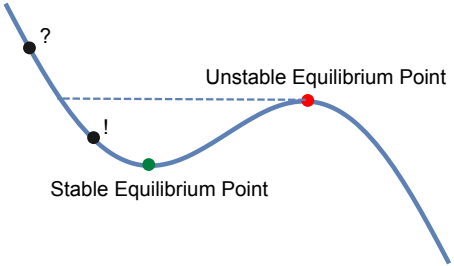
Energy method

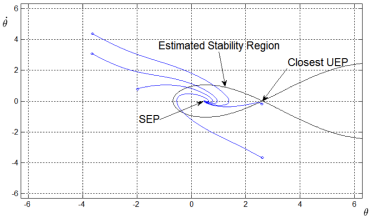




- If $E(\delta_0, \dot{\delta}_0) < E_{CUEP}$, then stable
- Fast transient stability certificate
- Computing E_{CUEP} is an NP-hard problem

Energy method





- If $E(\delta_0, \dot{\delta}_0) < E_{CUEP}$, then stable
- Fast transient stability certificate
- Computing E_{CUEP} is an NP-hard problem
- Certificates are generally conservative

Modeling Approach

- Non-linear swing equation

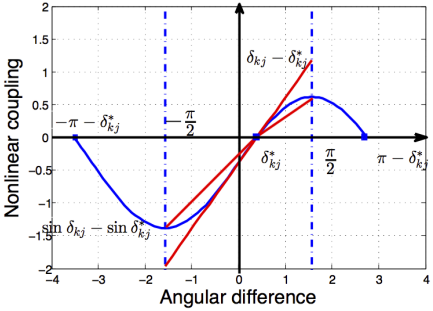
$$m_k\ddot{\delta}_k + d_k\dot{\delta}_k + \sum_{j\in\mathcal{N}_k} a_{kj}\sin(\delta_k - \delta_j) = P_k \tag{1}$$

$$m_k\ddot{\delta}_k + d_k\dot{\delta}_k + \sum_{j\in\mathcal{N}_k} a_{kj}(\sin(\delta_{kj}) - \sin(\delta^*_{kj})) = 0 \tag{2}$$

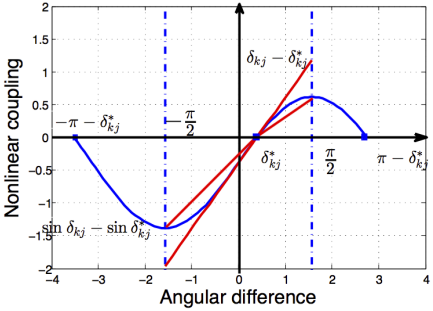
$$\dot{x} = Ax - BF(Cx) \tag{3}$$

- Structure-preserving model: A and B do not correspond to the reduced model
- $x = \delta_i - \delta_i^*$
- A, B, C are independent of the operating point P_k
- $F(Cx)$ stands for the non-linear function $sin(\delta_{kj}) - sin(\delta^*_{kj})$

Bounding nonlinearity



Bounding nonlinearity



- Sector bound on nonlinearity for polytope $\mathcal{P} : \{\delta, \dot{\delta} : |\delta_{kj}| < \frac{\pi}{2}\}$

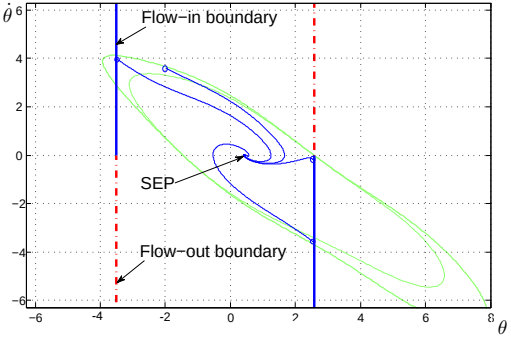
Stability certificate

- If:

$$\bar{A}^T P + P\bar{A} + \frac{(1-g)^2}{4} C^T C + PBB^T P \preceq 0 \tag{4}$$

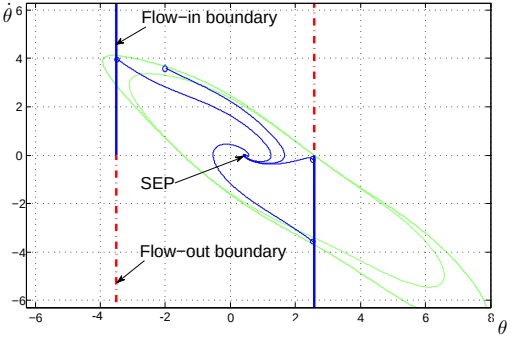
- there exists a quadratic Lyapunov function $V = x^T P x$ that is decreasing whenever $x(t) \in \mathcal{P}$.

Stability region



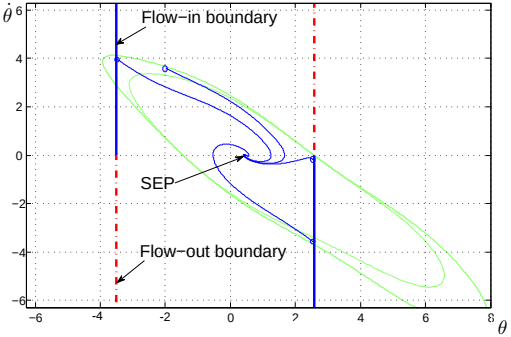
- Lyapunov function $x^T P x$ can be interpreted as an ellipsoid

Stability region



- Lyapunov function $x^T P x$ can be interpreted as an ellipsoid
- Due to the sector bound on the nonlinear $\sin()$ term, stability is certified only as long as we stay within $[-\pi/2, \pi/2]$

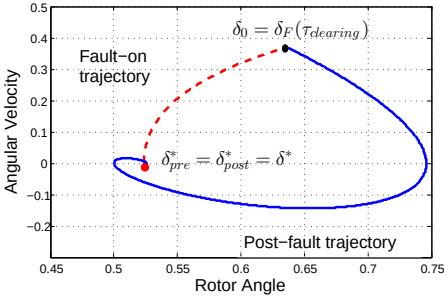
Stability region



- Lyapunov function $x^T P x$ can be interpreted as an ellipsoid
- Due to the sector bound on the nonlinear $\sin()$ term, stability is certified only as long as we stay within $[-\pi/2, \pi/2]$
- Finding the V_{min} within these bounds is now a convex problem!
  - We can solve (even large) convex problems fast and efficiently

Extensions to Remedial Actions

- Can incorporate inertia and damping control by appropriately changing A and B $\Rightarrow$ bound the growth of Lyapunov function



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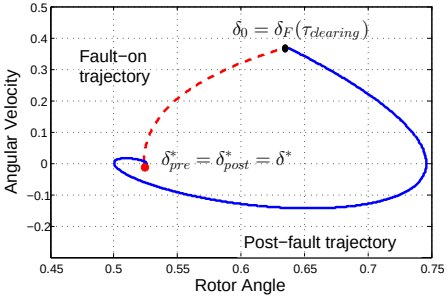
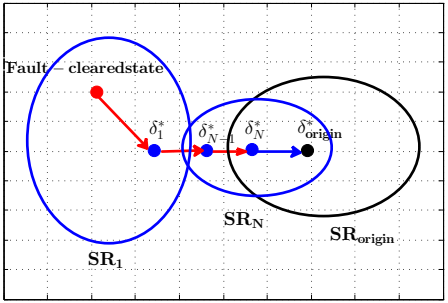


- Can incorporate topology control, e.g. FACTS, by appropriately changing A and $B \Rightarrow$ generate a set of ellipsoids that will guarantee the convergence of x_0 to the post-fault equilibrium



Chance-constrained AC Optimal Power Flow with Convex Relaxations

work with:
Andreas Venzke

Το πρόβλημα της βέλτιστης ροής ισχύος

ελαχιστοποίησε το κόστος ηλεκτρικής ενέργειας

λαμβάνοντας υπόψη:

τη ζήτηση των ηλεκτρικών φορτίων

τη μέγιστη ισχύ των γεννητριών

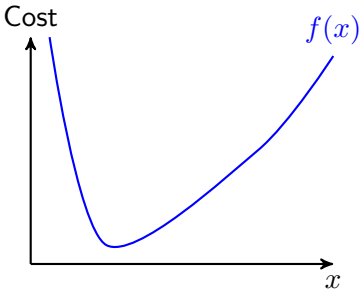
το μέγιστο ρεύμα που μπορούν να μεταφέρουν οι γραμμές μεταφοράς

τα όρια τάσης των ζυγών

- Το πρόβλημα ειναι:
  - μη γραμμικό: η ροή ισχύος εξαρτάται από τα τετράγωνα των τάσεων
  - μη κυρτό: υπάρχουν παραπάνω από ένα ελάχιστα

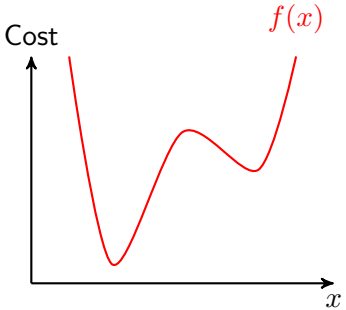
Convex vs. Non-convex Problem

Convex Problem



One global minimum

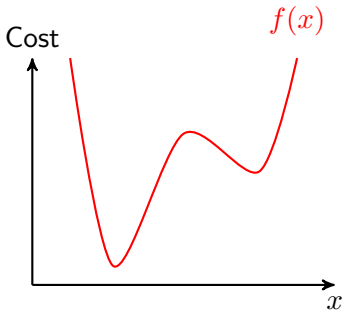
Non-convex problem



Several local minima

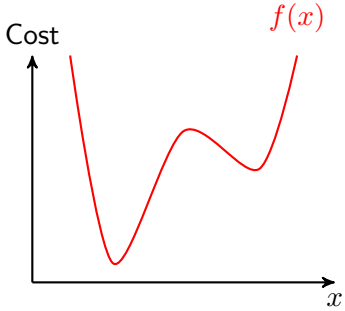
Several local minima: So what?

- Assume that the difference in the cost function of a local minimum versus a global minimum is 10%
- The total electric energy cost in the US is $\approx$ 400 Billion\$/year
- 10% amounts to 40 billion US\$ in economic losses per year
- Even 1% difference is huge
- Convex problems guarantee that we find a global minimum $\Rightarrow$ convexify the OPF problem



Convexifying the Optimal Power Flow problem (OPF)

- Convex relaxations transform the OPF to a convex Semi-Definite Program (SDP)



Convex Relaxation

[1] Javad Lavaei and Steven H Low. "Zero duality gap in optimal power flow problem". In: *IEEE Transactions on Power Systems* 27.1 (2012), pp. 92–107

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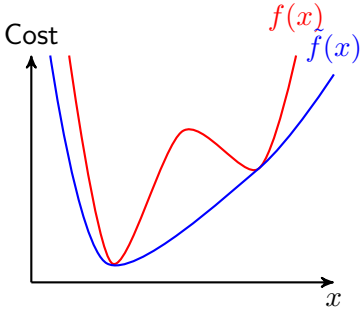

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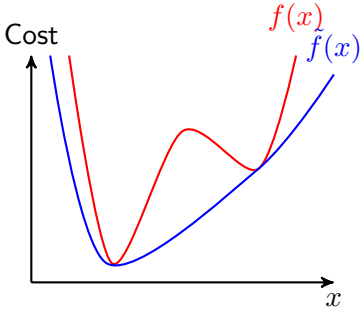
Convexifying the Optimal Power Flow problem (OPF)

- Convex relaxations transform the OPF to a convex Semi-Definite Program (SDP)
- Under certain conditions, the obtained solution is the global optimum to the original OPF problem[1]



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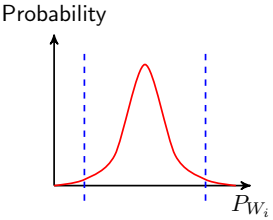
Introducing Uncertainty

- Increasing share of uncertain renewables
  $\Rightarrow$ Include chance constraints in OPF:
  Constraints should be fullfilled for a defined probability ϵ, given an underlying distribution of the uncertainty



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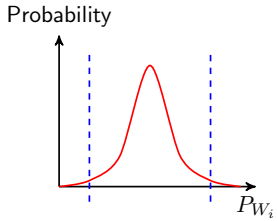
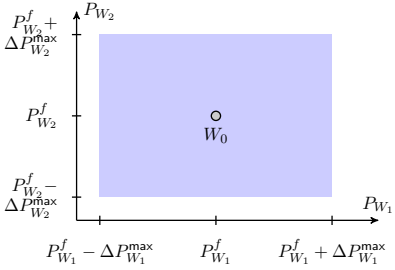


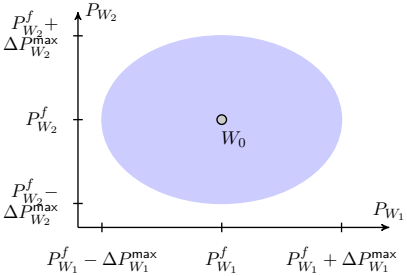
  - Uncertainty in wind forecast errors
  - Our Goal:
    - use the exact AC power flow equations,
    - in an elegant way: no iterations, no linearizations

Uncertainty Sets - Rectangular & Gaussian

How to model the uncertainty distribution of forecast errors ΔP_{W_i}?



Rectangular uncertainty set: Upper and lower bounds known.

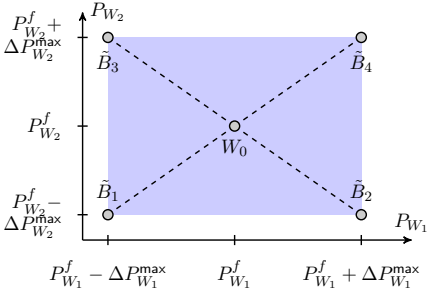


Ellipsoid uncertainty set: Multivariate Gaussian distribution.

Extend the work of (Vrakopoulou et al)[2] to: affine policies for multivariate sets, introduce gaussians, and ensure the existence of feasible solutions.

[2] Maria Vrakopoulou et al. "Probabilistic security-constrained AC optimal power flow". In: *IEEE PowerTech (POWERTECH)*. Grenoble, France, 2012.

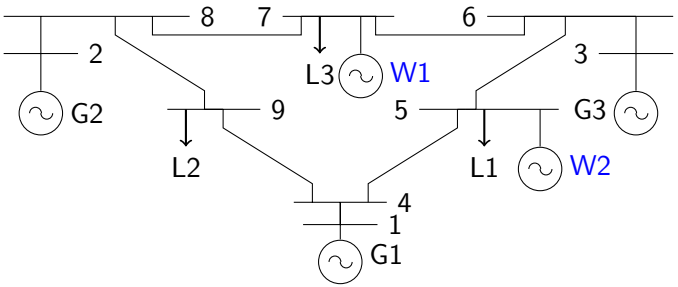
Formulation for Rectangular Uncertainty Set



- It suffices to enforce the chance constraints at the vertices v of the uncertainty set[3].

[3] Kostas Margellos, Paul Goulart, and John Lygeros. "On the road between robust optimization and the scenario approach for chance constrained optimization problems". In: *IEEE Transactions on Automatic Control* 59.8 (2014), pp. 2258–2263.

Test System



Modified IEEE 9-bus system with wind farms W1 and W2

- W1 with $\pm$ 50 MW deviation inside confidence interval
- W2 with $\pm$ 40 MW deviation inside confidence interval
- SDP-Solver: MOSEK
- Coded with Julia (open-source)

Simulation Results

Affine Policy for Rectangular Uncertainty Set

Generator droops	$d_1 =$ [0.5 0.25 0.25 0 -1 0 0 0 0]
Generator droops	$d_2 =$ [0.5 0.25 0.25 0 0 0 -1 0 0]
Weight power loss	$\mu = 0.4\ \frac{\$}{h\text{MW}}$
Generator cost	3378.73 $\frac{\$}{h}$
Eigenvalue ratios	$\rho(W_0) = 6.4 \times 10^6$
	$\rho^*(W_0 + \Delta\tilde{P}_1^{\max}\tilde{B}_1) = 2.5 \times 10^5$
	$\rho^*(W_0 + \Delta\tilde{P}_2^{\max}\tilde{B}_2) = 2.4 \times 10^5$
	$\rho^*(W_0 + \Delta\tilde{P}_3^{\max}\tilde{B}_3) = 2.7 \times 10^6$
	$\rho^*(W_0 + \Delta\tilde{P}_4^{\max}\tilde{B}_4) = 1.9 \times 10^6$

# Gen	V_G [p.u.]	P_G [MW]	Q_G [Mvar]	V_G^* [p.u.]	P_G^* [MW]	Q_G^* [Mvar]
G1	1.10	64.70	8.09	1.07	60.96	31.00
G2	1.09	97.21	-12.17	1.10	95.34	32.70
G3	1.08	65.43	-32.98	0.97	63.56	-80.45
W1	—	50.00	11.45	—	100.00	22.94
W2	—	40.00	1.39	—	0.00	0.00
$\sum$	—	317.34	-24.23	—	319.86	6.18

# Branch	from	to	P_{lm} [MW]	P_{lm}^* [MW]	Q_{lm} [Mvar]	Q_{lm}^* [Mvar]
3	5	6	42.87	67.50	-24.07	-35.04
Maximum voltage [p.u.]			$V^{\max}$	1.100	$(V^{\max})^*$	1.100

- we satisfy the conditions to obtain the global optimum
- all constraints are satisfied
- we find the true global minimum

Conclusions

- "Semidefinite programming is the most exciting development in mathematical programming in the 1990's"[4]
  - First applications in power systems in the 2010's

- Power interruptions are extremely costly; secure operation is challenging
  - SDP-based methods can extract less conservative stability certificates
- Large systems have high costs $\Rightarrow$ cannot afford to find a suboptimal local minimum
  - SDP-based optimization allows to recover the global optimum
  - we showed how to integrate wind uncertainty

- Challenges: Numerics & scalability

[4] Robert M. Freund. *Introduction to Semidefinite Programming*. MIT Lecture Notes. 2009.

Thank you!

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T.L. Vu and K. Turitsyn. "A Framework for Robust Assessment of Power Grid Stability and Resiliency". In: *IEEE Transactions on Automatic Control* PP.99 (2016), pp. 1–1. ISSN: 0018-9286. DOI: 10.1109/TAC.2016.2579743
S. Chatzivasileiadis, T.L. Vu, and K. Turitsyn. "Remedial Actions to Enhance Stability of Low-Inertia Systems". In: *IEEE Power and Energy Society General Meeting 2016, Boston, MA, USA*. July 2016, pp. 1–5
T.L. Vu, S. Chatzivasileiadis, H.D. Chang and K. Turitsyn. *Structural Emergency Control Paradigm*. Submitted. [Online]: arxiv.org/abs/1607.08183. 2016
A. Venzke, S. Chatzivasileiadis, et al. *Chance-Constrained Optimal Power Flow with Convex Relaxations*. to be submitted January 2017