Los Alamos Winter School and Conference

Physics-Informed Neural Networks for Power Systems

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Joint work with Georgios Misyris, Andreas Venzke
and with Guannan Qu, Steven Low

Why even think of applying Machine Learning in Energy Systems?

1. **Extremely fast**
   - computation within only a **few milliseconds** (100x – 1000x faster than conventional methods)

2. Good alternative if we do not have full knowledge of the actual model
   - Handle **very complex systems**
   - **Infer** from incomplete data

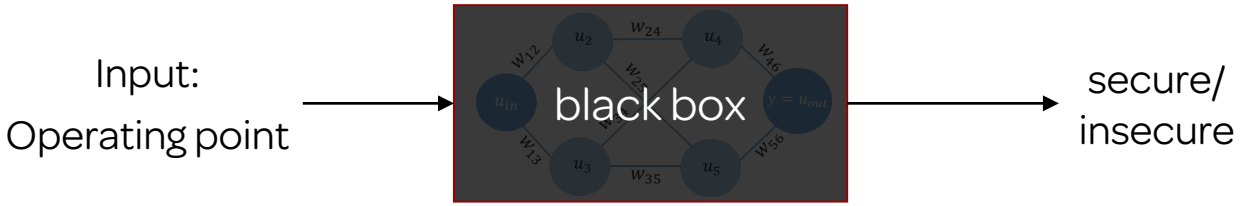
Energy systems: extremely large and extremely complex

Electric power grid: "the **largest machine** humans ever built"

But:

1. **Would an Operator ever trust AI in the Control Room?**
2. **Why use ML when engineers and scientists have spent the past 100 years developing good models for power systems?**

ML Barriers for Power systems

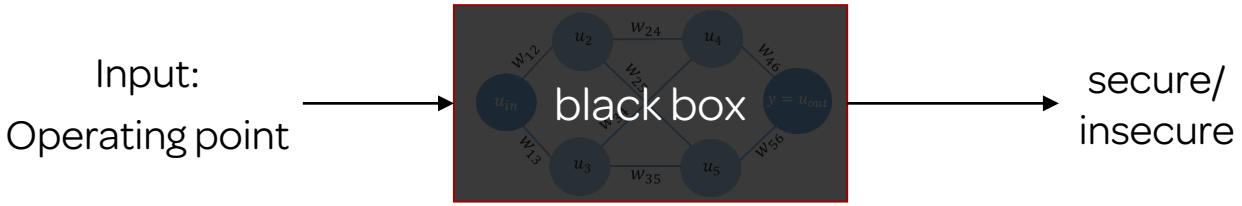


1. Why would we use a "**black box**" to decide about **a safety-critical application**?

2. **Accuracy is** a **purely statistical** performance metric. Who guarantees that the Neural Network can handle well previously unseen operating points?

3. Why would we only depend on **discrete and incomplete data**, when we have developed **detailed physical models** over the past 100 years?

ML Barriers for Power systems



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1. Integrate the power system models in **NN training**

→potential to replace conventional solvers?

2. Integrate the physical models in Neural Network verification : worst-case guarantees for the NN performance!

This talk

1. **Physics-Informed Neural Networks for Power System Dynamics**
   - Regression neural networks → estimation of numerical values such as rotor angle and frequency
   - Work inspired by Raissi et al* who applied it on physics problems
   - There exist a few recent works that use similar principles and apply PINNs on Optimal Power Flow problems (see Kekatos et al, K. Baker et al, Hentenryck et al, among others)

2. **Worst-case Guarantees for Neural Networks Learning OPF**
   - Introducing physical models to NN verification
   - Formulating MILPs that provide worst-case guarantees

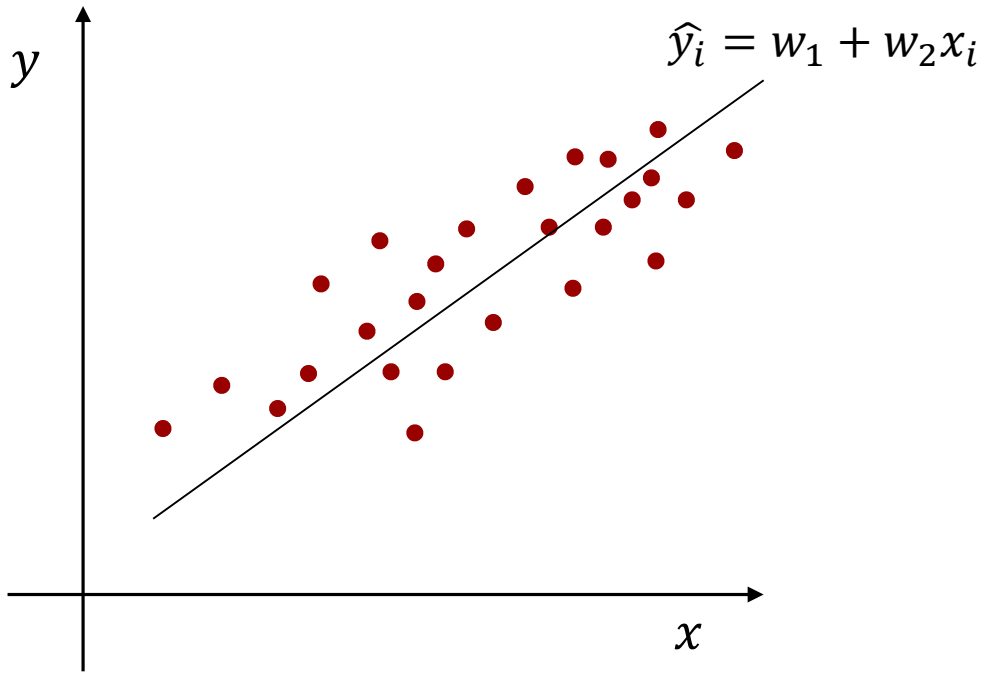
*M. Raissi, P. Perdikaris, and G. Karniadakis, Physics-Informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations", Journal of Computational Physics, vol.378, pp. 686-707, 2019

Physics-Informed Neural Networks for Power Systems

Neural Networks: An advanced form of non-linear regression

y_i: actual/correct value

$\hat{y}_i$: estimated value



Loss function: Estimate best w_1, w_2 to fit the training data

$$\min_{w_1, w_2} \quad \|y_i - \hat{y}_i\|$$

s.t.

$$\hat{y}_i = w_1 + w_2 x_i \qquad \forall i$$

Traditional training of neural networks required no information about the underlying physical model. Just data!

Physics Informed Neural Networks

- Automatic differentiation: derivatives of the neural network output with respect to the input can be computed during the training procedure
- A differential-algebraic model of a physical system can be included in the neural network training*
- Neural networks can now exploit knowledge of the actual physical system
- Machine learning platforms such as Tensorflow enable these capabilities

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Physics-Informed Neural Networks for Power Systems

"Original" Loss function

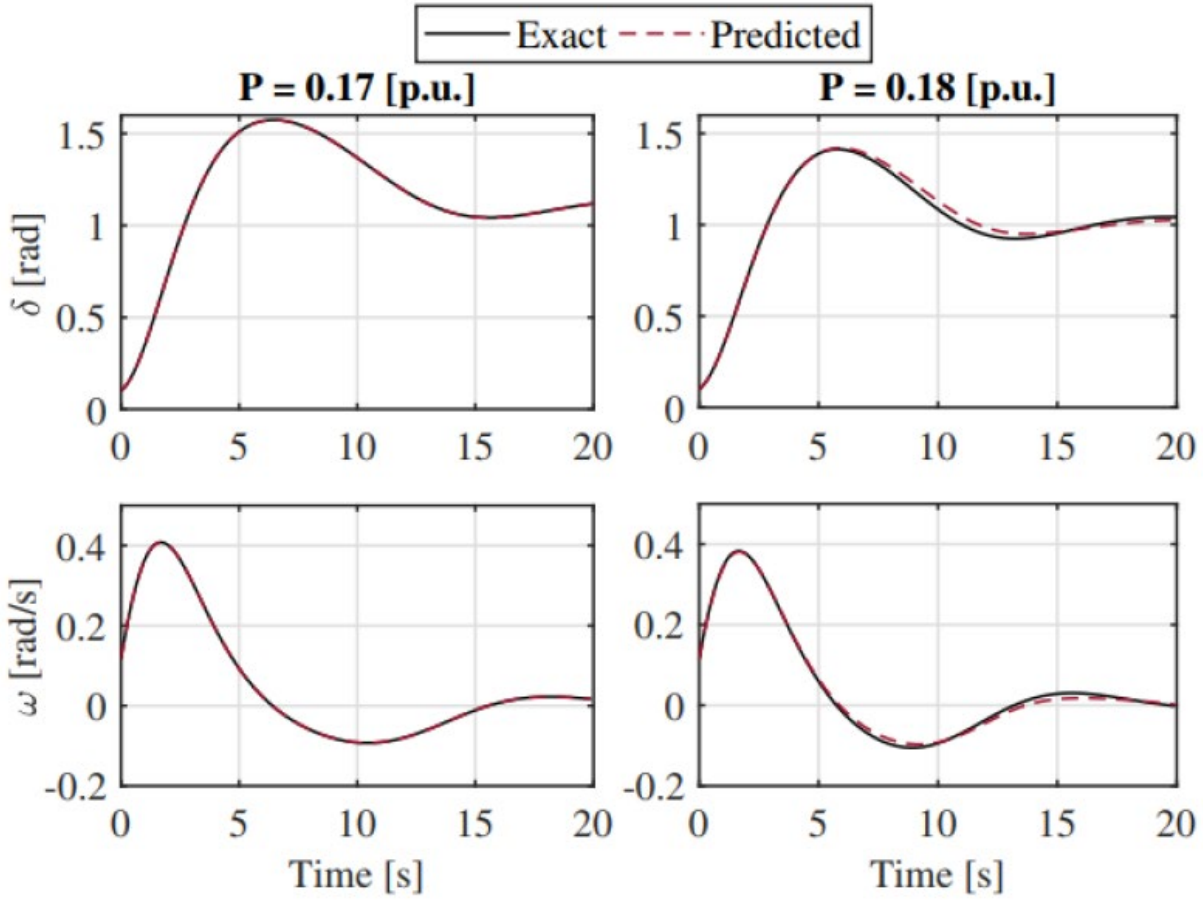
$$\min_{\mathbf{W},\mathbf{b}} \quad \frac{1}{|N_\delta|}\sum_{i\in N_\delta}|\hat{\delta}-\delta^i|^2+\frac{1}{|N_f|}\sum_{i\in N_f}|f(\hat{\delta})|^2 \tag{6a}$$

$$s.t. \qquad \hat{\delta}=NN(t,P_m,\mathbf{W},\mathbf{b}) \tag{6b}$$

$$\dot{\hat{\delta}}=\frac{\partial\hat{\delta}}{\partial t}, \qquad \ddot{\hat{\delta}}=\frac{\partial\dot{\hat{\delta}}}{\partial t} \tag{6c}$$

$$f(\hat{\delta})=M\ddot{\hat{\delta}}+D\dot{\hat{\delta}}+A\sin\hat{\delta}-P_m \tag{6d}$$

Swing equation



G. S. Misyris, A. Venzke, S. Chatzivasileiadis, Physics-Informed Neural Networks for Power Systems. Presented at the Best Paper Session of IEEE PES GM 2020. https://arxiv.org/pdf/1911.03737.pdf

Physics-Informed Neural Networks for Power Systems

"Original" Loss function

"Physics-Informed" term

$$\min_{\mathbf{W},\mathbf{b}} \quad \frac{1}{|N_\delta|}\sum_{i\in N_\delta}|\hat{\delta}-\delta^i|^2+\frac{1}{|N_f|}\sum_{i\in N_f}|f(\hat{\delta})|^2 \tag{6a}$$

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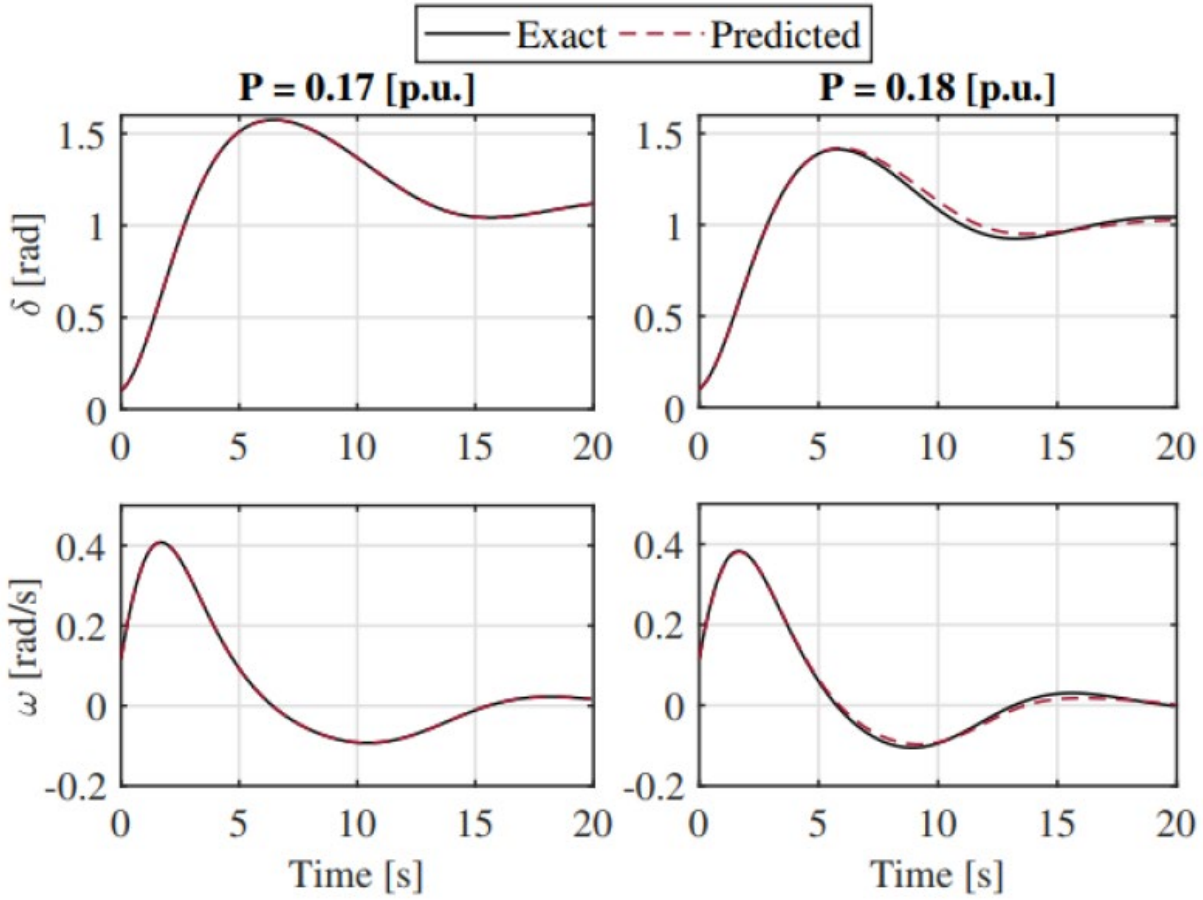

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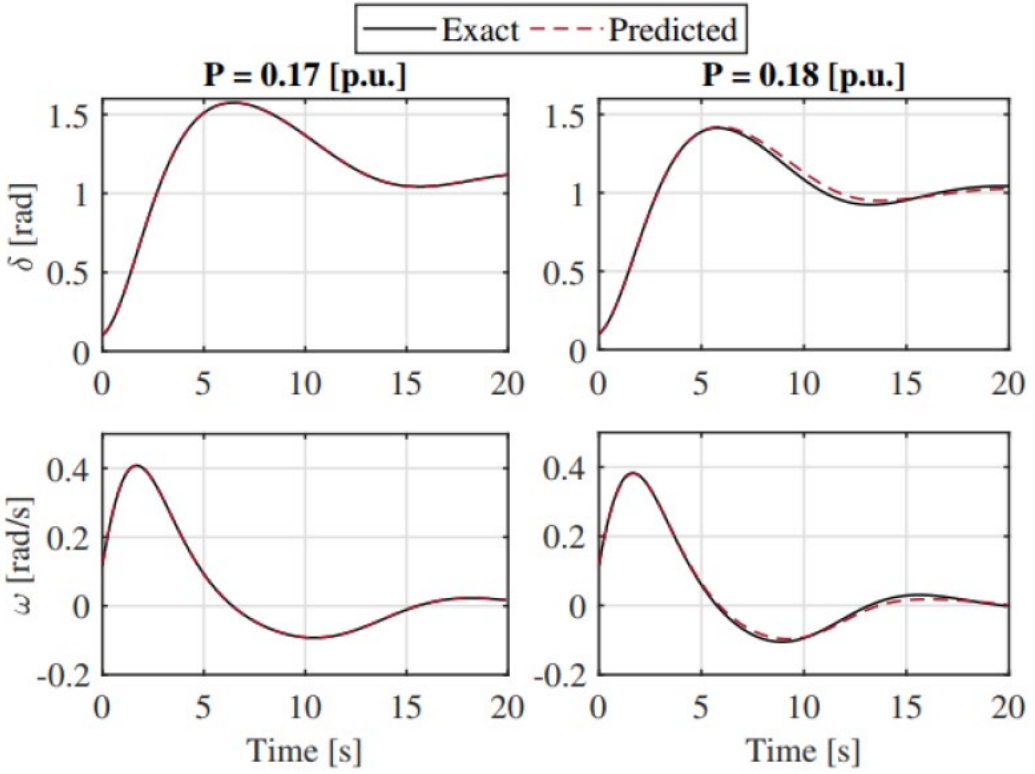
Physics-Informed Neural Networks for Power Systems

- Physics-Informed Neural Networks (PINN) **can potentially replace** solvers for systems of differential-algebraic equations
- In our example: PINN 87 times faster than ODE solver
- Can **directly estimate** the rotor angle at **any** time instant



Code is available on GitHub: https://github.com/gmisy/Physics-Informed-Neural-Networks-for-Power-Systems/

G. S. Misyris, A. Venzke, S. Chatzivasileiadis, Physics-Informed Neural Networks for Power Systems. Presented at the Best Paper Session of IEEE PES GM 2020. https://arxiv.org/pdf/1911.03737.pdf

Current Work and Open Challenges

- PINNs for System Identification

- PINNs for quick simulation/estimation of converter dynamics
  - Can provide a quick estimate to a multi time-scale or co-simulation platform

- Tractability is an issue
  - Currently capturing all system dynamics for up to an 11-bus system with a single PINN
  - PINNs can also be applied for larger systems, but the computing effort (e.g. for training) will increase as well
  - Need to develop good approaches to maintain a lower computing effort while we scale up to bigger systems

Learning OPF:
Worst-case Guarantees for Regression Neural Networks

A. Venzke, G. Qu, S. Low, S. Chatzivasileiadis, Learning Optimal Power Flow: Worst-case Guarantees for Neural Networks. **Best Student Paper Award** at IEEE SmartGridComm 2020. https://arxiv.org/pdf/2006.11029.pdf

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Quick Reminder: DC Optimal Power Flow

- **Objective**: find the minimum cost generation dispatch
- **Input**: Varying load demand at different nodes

- Considered constant: generator costs; system topology

$$
\begin{aligned}
\min_{\mathbf{p}_g, \boldsymbol{\theta}} \quad & \mathbf{c}^T \mathbf{p}_g && \text{Minimizes generation cost} \\
\text{s.t.} \quad & \mathbf{M}_g \mathbf{p}_g - \mathbf{M}_d \mathbf{p}_d = \mathbf{B}_{bus} \boldsymbol{\theta} && \text{Nodal power balance} \\
& -\mathbf{p}_{line}^{max} \leq \mathbf{B}_{line} \boldsymbol{\theta} \leq \mathbf{p}_{line}^{max} && \text{Transmission line limits} \\
& \mathbf{p}_g^{min} \leq \mathbf{p}_g \leq \mathbf{p}_g^{max} && \text{Generator limits}
\end{aligned}
$$

Quick Reminder: DC Optimal Power Flow

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Several recent approaches in the literature that apply Neural Networks for solving the DC-OPF

- Demonstrate up to **100x speedup**
- But **no performance guarantees**

$$
\begin{aligned}
\min_{\mathbf{p}_g, \boldsymbol{\theta}} \quad & \mathbf{c}^T \mathbf{p}_g && \text{Minimizes generation cost} \\
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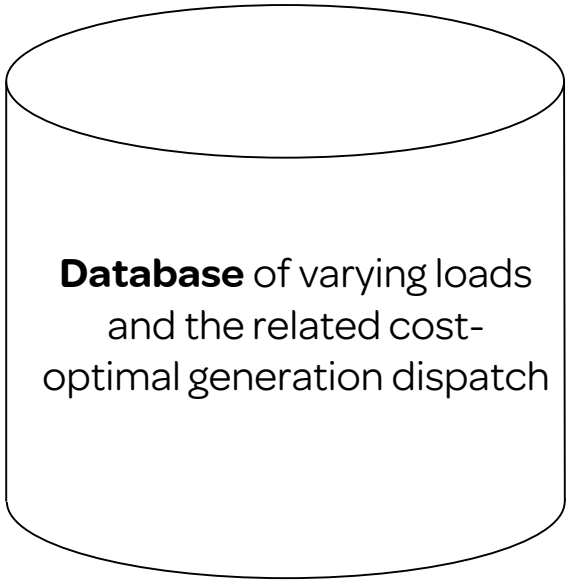
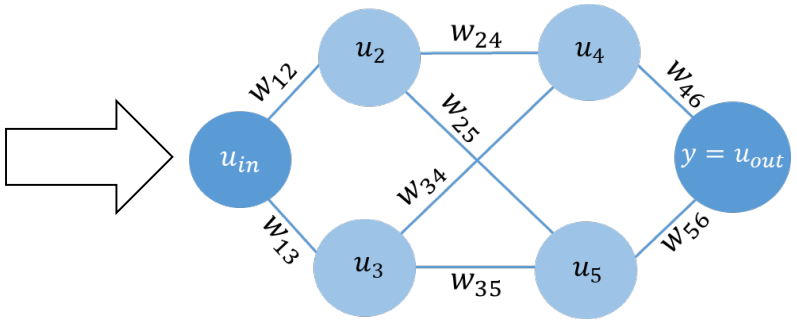
\end{aligned}
$$

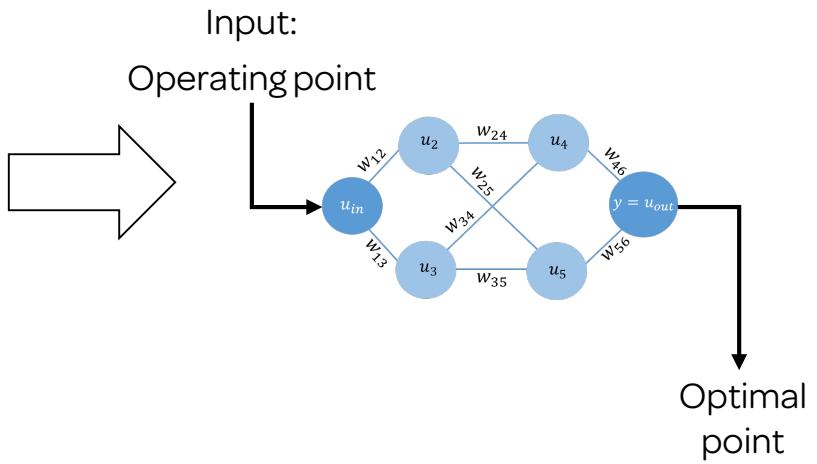
Guiding Application for Regression NN: Learning OPF

Approaches proposed up to now





5. Use the NN



1. Split the database in a training set and a test set

2. Train a neural network

3. Test the neural network

4. Is accuracy high enough?

NN Output:

Optimal generation dispatch

Extremely fast:

up to 100x faster

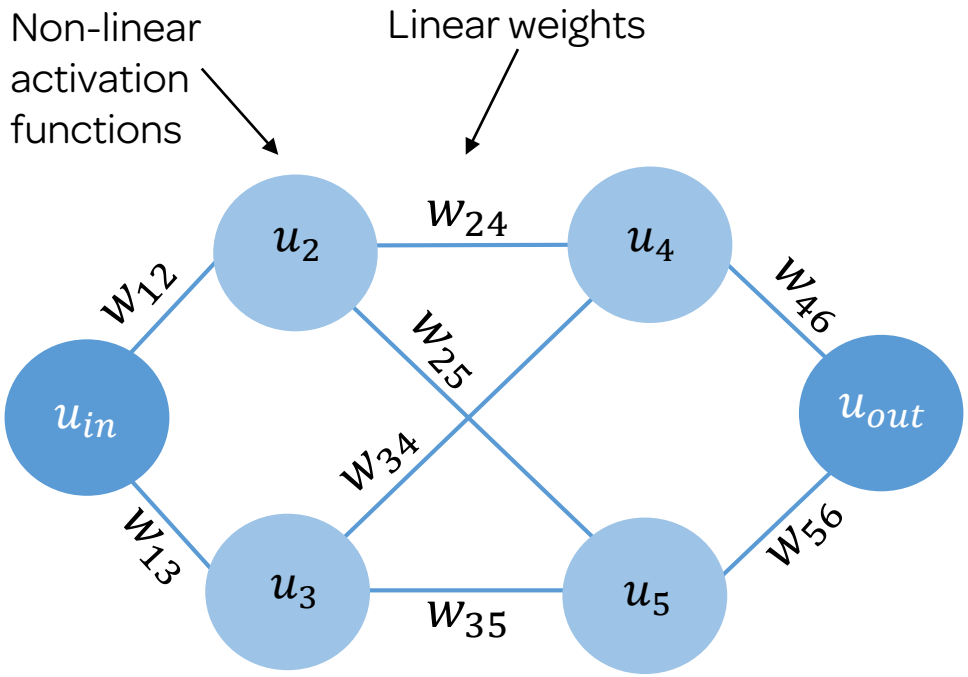
Neural Network Verification for Regression NNs: HOW?

1. **Exact transformation:** Convert the neural network to a **set of linear equations with binaries**
   - The Neural Network can be included in a mixed-integer linear program

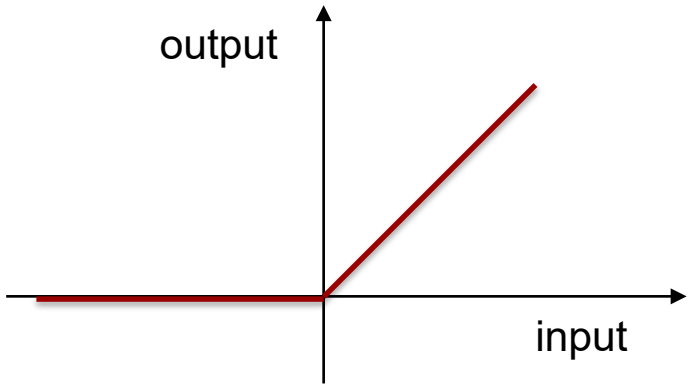
2. Include the physical equations

3. Formulate an **optimization** problem (MILP) and solve it → certificate for NN behavior

From Neural Networks to Mixed-Integer Linear Programming



- Most usual activation function: ReLU
- ReLU: Rectifier Linear Unit



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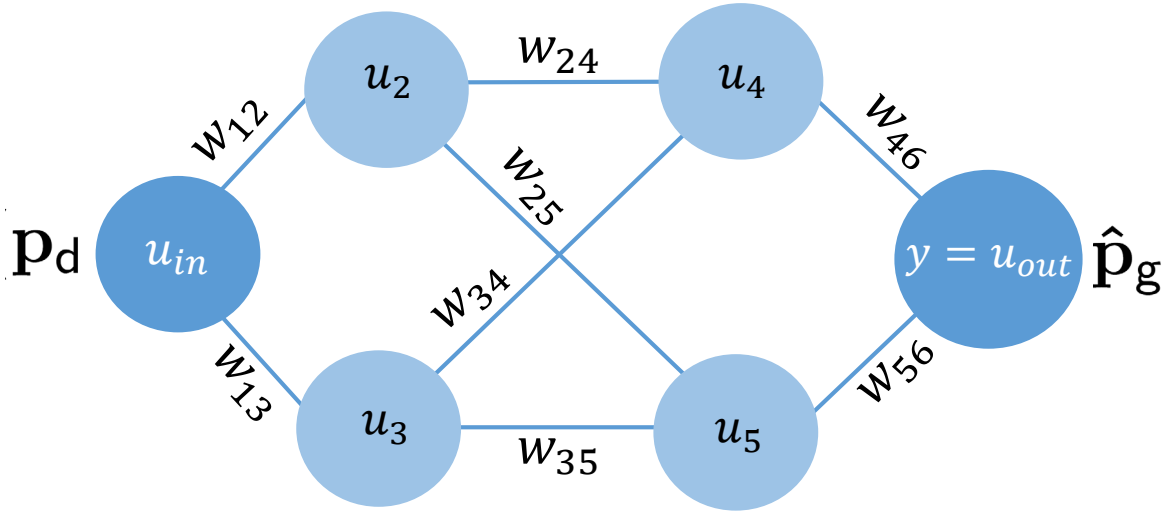

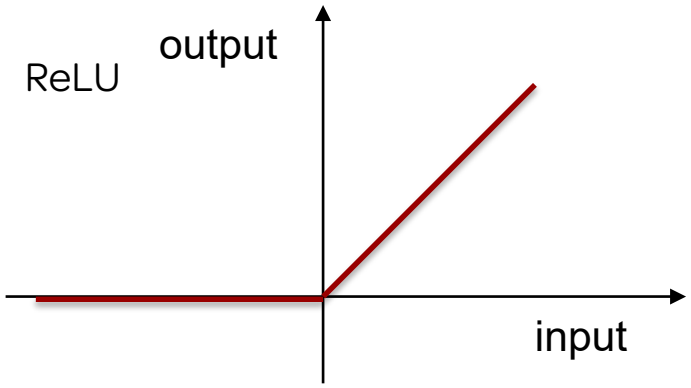

- Linear weights
- On every node: a non-linear activation function
  - ReLU: $\boldsymbol{u_j = \max\left(0, w_{ij}u_i + b_i\right)}$
- But ReLU can be transformed to a piecewise linear function with binaries

MILP → $\hat{\mathbf{p}}_\mathrm{g} = NN(\mathbf{p}_\mathrm{d})$

Part I: Maximum limit-violations

1. Maximum violation of generator limits

$$\nu_{\mathrm{g}} = \max(\hat{\mathbf{p}}_{\mathrm{g}} - \mathbf{p}_{\mathrm{g}}^{\max}, \mathbf{p}_{\mathrm{g}}^{\min} - \hat{\mathbf{p}}_{\mathrm{g}}, \mathbf{0})$$

$$\begin{aligned} \max \quad & \nu_{\mathrm{g}} \\ \text{s.t.} \quad & \mathbf{A}_{\mathrm{d}}\mathbf{p}_{\mathrm{d}} \leq \mathbf{b}_{\mathrm{d}} && \text{Convex polytope as input domain } \mathcal{D} \\ & \hat{\mathbf{p}}_{\mathrm{g}} = NN(\mathbf{p}_{\mathrm{d}}) && \text{Mixed-integer reformulation of trained NN} \end{aligned}$$

Example:

$$0.6\,\boldsymbol{p}_{\mathrm{d}}^{\max} \leq \boldsymbol{p}_{\mathrm{d}} \leq 1.0\,\boldsymbol{p}_{\mathrm{d}}^{\max}$$

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Example:

$$0.6\,\boldsymbol{p}_{\mathrm{d}}^{\max} \leq \boldsymbol{p}_{\mathrm{d}} \leq 1.0\,\boldsymbol{p}_{\mathrm{d}}^{\max}$$

2. Maximum violation of line limits

$$\nu_{\mathrm{line}} = \max(|\mathbf{B}_{\mathrm{line}}\tilde{\mathbf{B}}_{\mathrm{bus}}^{-1}(\mathbf{M}_{\mathrm{g}}\hat{\mathbf{p}}_{\mathrm{g}} - \mathbf{M}_{\mathrm{d}}\mathbf{p}_{\mathrm{d}})^{\mathrm{nsb}}| - \mathbf{p}_{\mathrm{line}}^{\max}, \mathbf{0})$$

Line flow equations for
DC-OPF based on PTDFs

$$\begin{aligned} \max \quad & \nu_{\mathrm{line}} \\ \text{s.t.} \quad & \mathbf{A}_{\mathrm{d}}\mathbf{p}_{\mathrm{d}} \leq \mathbf{b}_{\mathrm{d}} && \text{Convex polytope as input domain } \mathcal{D} \\ & \hat{\mathbf{p}}_{\mathrm{g}} = NN(\mathbf{p}_{\mathrm{d}}) && \text{Mixed-integer reformulation of trained NN} \end{aligned}$$

Worst violation over the **whole training dataset** (training+test set)

Our algorithm: **provable** worst-case guarantee over the **whole input domain**

	Empirical lower bound		Exact worst-case guarantee	
Test cases	ν_{g} (MW)	ν_{line} (MW)	ν_{g} (MW)	ν_{line} (MW)
case9				
case30				
case39				
case57				
case118				
case162				
case300				

ν_{g} Maximum violation of generator limits

ν_{line} Maximum violation of line limits

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case9	2.5	1.8	2.8	1.9
case30	1.7	0.6	3.6	3.1
case39	51.9	37.2	270.6	120.0
case57	4.2	0.0	23.7	0.0
case118	149.4	15.6	997.8	510.8
case162	228.0	180.0	1563.3	974.1
case300	474.5	692.7	3658.5	3449.3

ν_{g} Maximum violation of generator limits

ν_{line} Maximum violation of line limits

Over the whole input domain **violations can be much larger** (here ~7x) compared to what has been estimated empirically on the dataset

Worst violation over the **whole training dataset** (training+test set)

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Our method provides **guarantees that no NN output will violate the line limits** over the whole input domain

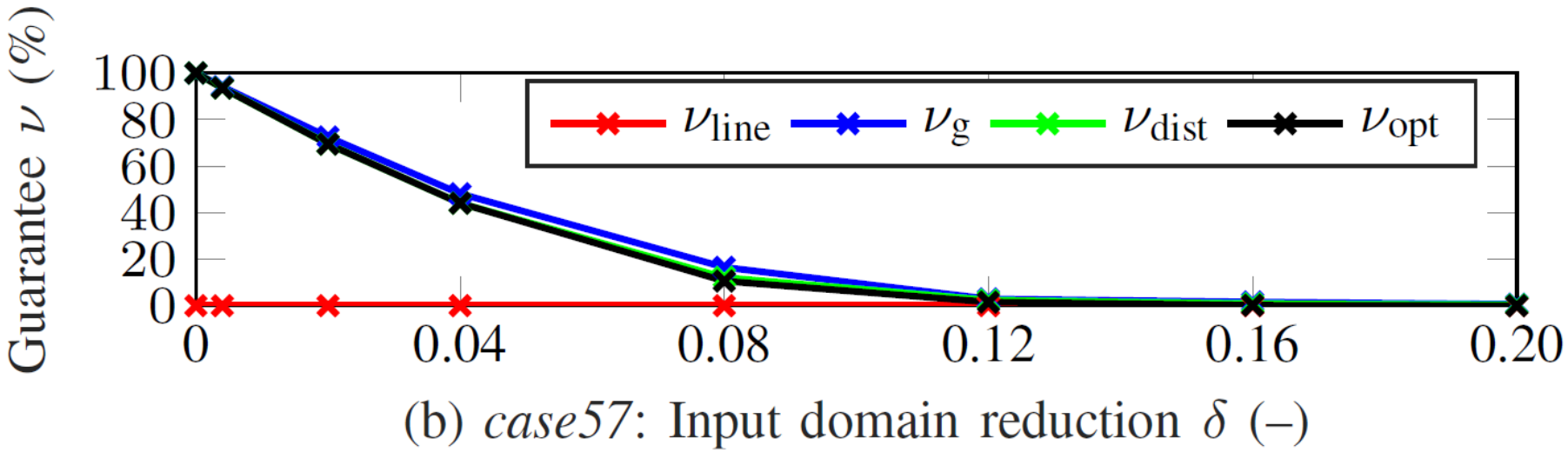
How can we reduce the worst-case violations?

- From our experiments with DC-OPF in 7 different test power systems, we observed that the **worst-case violations occur at the boundary of the input domain**

- Possible solution:
  1. Train on a larger input domain
  2. Use the NN on a subdomain of the original training input

Reducing the worst-case violations

100% on y-axis = Worst-case violation for $\delta = 0$



(b) *case57*: Input domain reduction δ (–)

- Input domain used for training

$$0.6\,\boldsymbol{p}_{\text{d}}^{\text{max}} \leq \boldsymbol{p}_{\text{d}} \leq 1.0\,\boldsymbol{p}_{\text{d}}^{\text{max}}$$

- Input domain for using the NN (and where worst-case violations were evaluated)

$$(0.6+\delta)\mathbf{p}_{\text{d}}^{\text{max}} \leq \mathbf{p}_{\text{d}} \leq (1.0-\delta)\mathbf{p}_{\text{d}}^{\text{max}}$$

Example: If $\delta = 0.1$ then $0.7\,\boldsymbol{p}_{\text{d}}^{\text{max}} \leq \boldsymbol{p}_{\text{d}} \leq 0.9\,\boldsymbol{p}_{\text{d}}^{\text{max}}$

A. Venzke, G. Qu, S. Low, S. Chatzivasileiadis, Learning Optimal Power Flow: Worst-case Guarantees for Neural Networks. **Best Student Paper Award** at IEEE SmartGridComm 2020.[.pdf | slides | video]

Wrap-up

- **Physics-Informed Neural Networks for Power System Dynamics**
  - Reduce dependence on discrete or incomplete data
  - Drastically reduce computing time
  - Potential to replace conventional differential-algebraic solvers?
    (maybe for use cases where computing time is more important than high accuracy?)

- **Provable worst-case guarantees for Regression Neural Networks**
  - Combination of Neural Network verification with underlying physical models
  - Application in OPF (and probably any linear program)

Thank you!

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Some code available at:

www.chatziva.com/downloads.html