Convex Relaxations of AC Optimal Power Flow under Uncertainty

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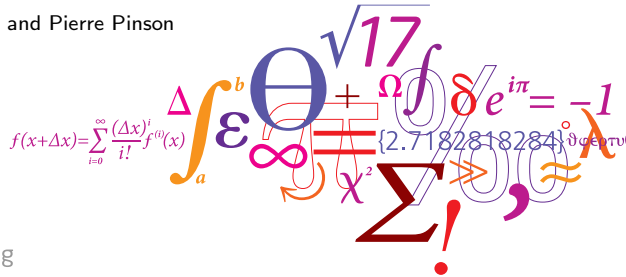


DTU Electrical Engineering
Department of Electrical Engineering

DTU Center for Electric Power and Energy

- Part of the Department of Electrical Engineering
- Among the strongest university centers in Europe with $\approx$ 100 employees
- Direct support from Siemens, Danfoss, and other big Danish companies
- Denmark has the highest proportion of wind power in the world (**44%** in 2017)
- DTU ranks in the top 10 universities in the world in Energy Science and Engineering in Shanghai Ranking (2015, 2016)

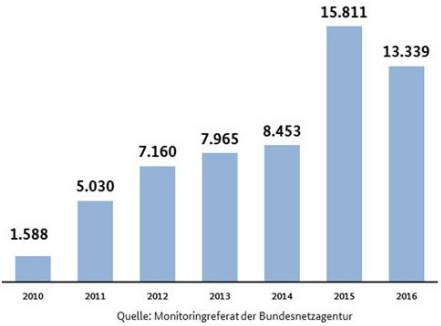
Source: Google Maps

Outline

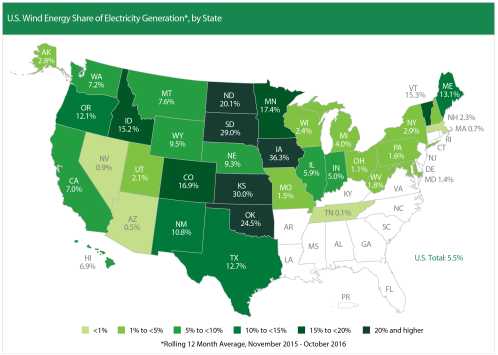
1. **Motivation and Scope of Our Work**
2. Convex Relaxations of Chance Constrained AC OPF
3. N-1 Security and Scalability
4. Interconnected AC and HVDC Grids
5. Conclusions

Why Consider Uncertainty?

Development of redispatch measures in German transmission grid (2016: 31.5 % RES)



Source: Bundesnetzagentur



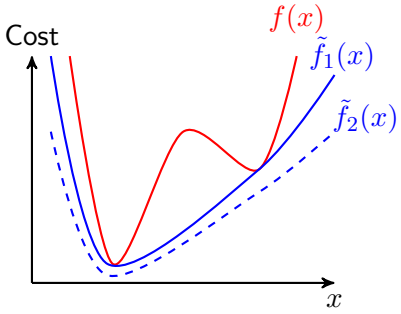
Source: AWEA

$\Rightarrow$ New tools necessary for power system operation of AC grids under uncertainty which are able to:

- anticipate forecast errors to maintain a secure system operation
- define a-priori suitable corrective control policies

Why Convex Relaxations?

- AC optimal power flow problem non-linear & non-convex
  - No guarantee obtained solution is global optimum
  - Distance to global optimum cannot be specified (cost)
- Semidefinite relaxation transforms AC-OPF to convex semi-definite program (SDP)



$\Rightarrow$ Under certain conditions, obtained solution is the global optimum to the original AC-OPF (Zero relaxation gap in work by Lavaei and Low[1])

[1] Javad Lavaei and Steven H Low. "Zero duality gap in optimal power flow problem". In: *IEEE Transactions on Power Systems* 27.1 (2012), pp. 92–107

Related Work

To address uncertainty, include chance constraints in the optimal power flow (OPF) problem, which define a maximum probability of violation.

To achieve a tractable formulation, related works in literature use

- an a-priori computed linearization of the power flow equations (e.g. DC-OPF)
- iteratively linearize the power flow equations depending on operating point

Linear vs. Iterative Chance Constrained OPF

Linear OPF

- Pros:
  - Faster
  - Scalable
- Cons:
  - No losses
  - No reactive power flows
  - Approximate

[1]Vrakopoulou, Margellos, Lygeros, Andersson, TPWRS 2013
[2]Bienstock, Chertkov, Harnett, SIAM Review 2014
[3]Lubin, Dvorkin, Backhaus, TPWRS, 2016

Iterative non-linear OPF

- Pros:
  - Losses and reactive power considered
  - Scalable
- Cons:
  - Non-convex $\rightarrow$ might be trapped in a local minimum
  - Convergence

[1]Zhang, Li, TPWRS, 2011
[2]Schmidli, Roald, Chatzivasileiadis, Andersson, PES GM 2016
[3]Roald, Andersson, TPWRS 2017

Scope of Our Work

- Integrate the chance constraints in the semidefinite relaxation of the AC-OPF for meshed transmission networks
- Develop a tractable OPF formulation which includes uncertainty in power injections and which allows to
  - include the full AC-OPF formulation (consider large uncertainty deviations)
  - corrective control policies related to active and reactive power, and voltage
  - provide (near-)global optimality guarantees
- First steps taken in Vrakopoulou et al.[2]

[2] Maria Vrakopoulou et al. "Probabilistic security-constrained AC optimal power flow". In: *IEEE PowerTech*. Grenoble, France, 2012

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2. **Convex Relaxations of Chance Constrained AC OPF**
   - OPF formulation
   - Including chance constraints
   - Tractable formulation for Gaussian and Robust Uncertainty Set
   - Investigating relaxation gap
   - IEEE 118 bus test case
3. N-1 Security and Scalability
4. Interconnected AC and HVDC Grids
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Optimal Power Flow Formulation

Introducing the variable transformation of complex bus voltages V:

$$\mathbf{X} := [\Re\{V\}\, \Im\{V\}]^T$$
$$\mathbf{W} = \mathbf{X}\mathbf{X}^T$$

$\Rightarrow$ The $2n_{\text{bus}}$ - dimensional vector $\mathbf{X}$ is transformed to $2n_{\text{bus}} \times 2n_{\text{bus}}$ - dimensional matrix $\mathbf{W}$

Optimal Power Flow Formulation

...for each node k and line lm:

Minimize Generation Cost	$\sum_{k\in\mathcal{G}}\{c_{k2}(\mathsf{Tr}\{\mathbf{Y}_k W\}+P_{D_k})^2 + c_{k1}(\mathsf{Tr}\{\mathbf{Y}_k W\}+P_{D_k})+c_{k0}\}$
s.t. Active Power Balance	$P_k^{\min} \leq \mathsf{Tr}\{\mathbf{Y}_k W\} \leq P_k^{\max}$
Reactive Power Balance	$Q_k^{\min} \leq \mathsf{Tr}\{\bar{\mathbf{Y}}_k W\} \leq Q_k^{\max}$
Bus Voltages	$(V_k^{\min})^2 \leq \mathsf{Tr}\{M_k W\} \leq (V_k^{\max})^2$
Active Branch Flow	$-P_{lm}^{\max} \leq \mathsf{Tr}\{\mathbf{Y}_{lm} W\} \leq P_{lm}^{\max}$
Apparent Branch Flow	$\mathsf{Tr}\{\mathbf{Y}_{lm} W\}^2 + \mathsf{Tr}\{\bar{\mathbf{Y}}_{lm} W\}^2 \leq (S_{lm}^{\max})^2$
Decomposition	$W = \underbrace{[\Re\{V\}\ \Im\{V\}]^T}_{X}\underbrace{[\Re\{V\}\ \Im\{V\}]}_{X^T}$

Matrices $\mathbf{Y}_k$, $\bar{\mathbf{Y}}_k$, $\mathbf{Y}_{lm}$ and $\bar{\mathbf{Y}}_{lm}$ are auxiliary variables resulting from the admittance matrix $\mathbf{Y}$ of the system.

Optimal Power Flow Formulation

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Semi-Definiteness of W	$W \succeq 0$
Rank Constraint	$\mathsf{rank}(W) = 1$

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Semi-Definiteness of W	$W \succeq 0$
Rank Constraint	$\cancel{\mathsf{rank}(W)=1} \Rightarrow$ Convex Relaxation

$\Rightarrow$ Lavaei and Low show that for the common IEEE test systems, the relaxation achieves zero relaxation gap, i.e. it is exact.

Optimal Power Flow Formulation

...for each node k and line lm:

Minimize Generation Cost	$\sum_{k\in\mathcal{G}}\{c_{k2}(\mathsf{Tr}\{\mathbf{Y}_k W\} + P_{D_k})^2 + c_{k1}(\mathsf{Tr}\{\mathbf{Y}_k W\} + P_{D_k}) + c_{k0}\}$
s. t. Active Power Balance	$P_k^{\min} \leq \mathsf{Tr}\{\mathbf{Y}_k W\} \leq P_k^{\max}$
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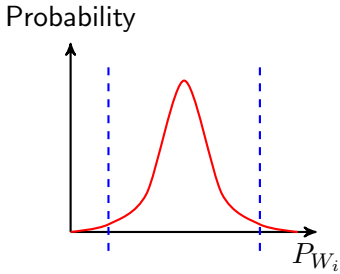
Semi-Definiteness of W	$W \succeq 0$

$\Rightarrow$ Note that most constraints are linear in W; generation cost and apparent branch flow are reformulated as SOC constraints; positive semi-definiteness of W.

Including Chance Constraints

- Include chance constraints in OPF: Constraints should be satisfied for a defined probability $1 - \epsilon$, given an underlying distribution of the uncertainty
- Include wind in-feeds with forecast value P_W^f and forecast error ζ: $P_{W_i} = P_{W_i}^f - \zeta_i$



Chance Constrained AC-OPF Formulation

...for each node k and line lm:

Minimize Generation Cost	$\sum_{k\in\mathcal{G}}\{c_{k2}(\mathsf{Tr}\{\mathbf{Y}_k W_0\}+P_{D_k})^2+ c_{k1}(\mathsf{Tr}\{\mathbf{Y}_k W_0\}+P_{D_k})+c_{k0}\}$
s. t.	OPF constraints for $W=W_0$
$\mathcal{P}\{$ Active Power Balance	$P_k^{\min}\leq \mathsf{Tr}\{\mathbf{Y}_k W(\zeta)\}\leq P_k^{\max}$
Reactive Power Balance	$Q_k^{\min}\leq \mathsf{Tr}\{\bar{\mathbf{Y}}_k W(\zeta)\}\leq Q_k^{\max}$
Bus Voltages	$(V_k^{\min})^2\leq \mathsf{Tr}\{M_k W(\zeta)\}\leq (V_k^{\max})^2$
Active Branch Flow	$-P_{lm}^{\max}\leq \mathsf{Tr}\{\mathbf{Y}_{lm} W(\zeta)\}\leq P_{lm}^{\max}$
Apparent Branch Flow	$\mathsf{Tr}\{\mathbf{Y}_{lm} W(\zeta)\}^2+\mathsf{Tr}\{\bar{\mathbf{Y}}_{lm} W(\zeta)\}^2\leq (S_{lm}^{\max})^2$
Semi-Definiteness of W	$W(\zeta)\succeq 0 \quad \}\geq 1-\epsilon$

W_0: Forecasted system state

Including Chance Constraints

- System state $W(\zeta)$ is a function of forecast errors ζ
- The resulting chance constrained problem is infinite-dimensional problem
  $\Rightarrow$ Chance constraints are intractable!

How can we achieve tractability?

- How to model the forecast error?
  $\Rightarrow$ Either using scenario-based methods or assuming probability distributions
- How to approximate system state $W(\zeta)$ as a function of forecast error?
  $\Rightarrow$ Piecewise affine approximation

Randomized and Robust Optimization

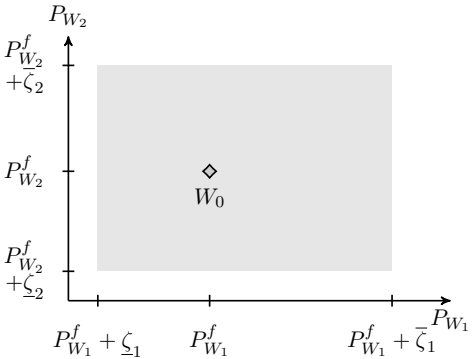
- No assumptions on the underlying distribution of the forecast errors.
- Then, we have to take at least N_s samples to enclose $1-\epsilon$ of the uncertainty set with confidence parameter β

$$N_{\mathsf{s}} \geq \frac{1}{1-\epsilon}\frac{e}{e-1}(\ln\frac{1}{\beta} + 2n_W - 1)$$

where e is Euler's number and n_W the number of wind farms.[3]

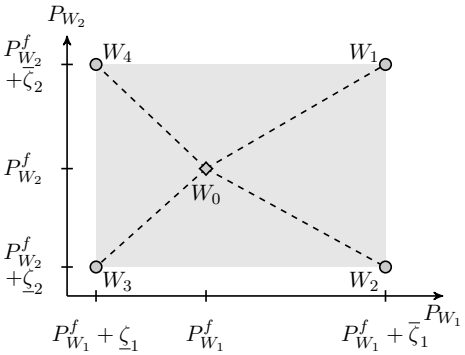
[3] Maria Vrakopoulou et al. "Probabilistic security-constrained AC optimal power flow". In: *IEEE PowerTech*. Grenoble, France, 2012

Randomized and Robust Optimization



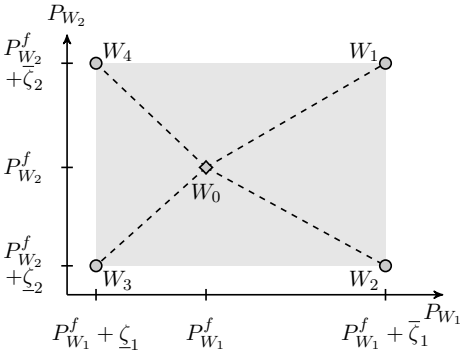
- The minimum and maximum bounds on the forecast errors $\zeta_i \in [\underline{\zeta}_i, \overline{\zeta}_i]$ are retrieved by a simple sorting operation among the N_s scenarios:

Randomized and Robust Optimization



- We use a piecewise affine policy which interpolates system state between forecasted system state W_0 and vertices of the uncertainty set W_{1-4}.
- That is, we compute the exact AC-OPF solution at each of the vertices and at the forecasted system state.

Randomized and Robust Optimization

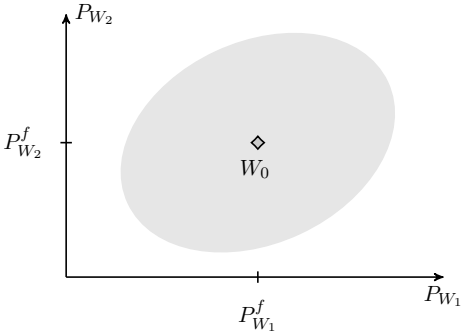


- As result of piecewise affine approximation, chance constraints are convex.
- Using robust optimization[3], it is sufficient to enforce chance constraints at the vertices of the uncertainty set.

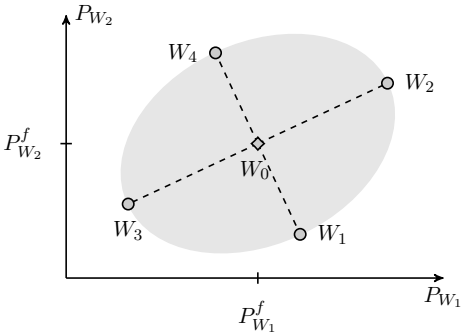
[3] Kostas Margellos, Paul Goulart, and John Lygeros. "On the road between robust optimization and the scenario approach for chance constrained optimization problems". In: *IEEE Transactions on Automatic Control* 59.8 (2014), pp. 2258–2263

Gaussian Uncertainty Set

- Here, we consider the individual (not joint) chance constraint violation probability.
- Assuming that the wind forecast errors are Gaussian distributed and possibly correlated yields:

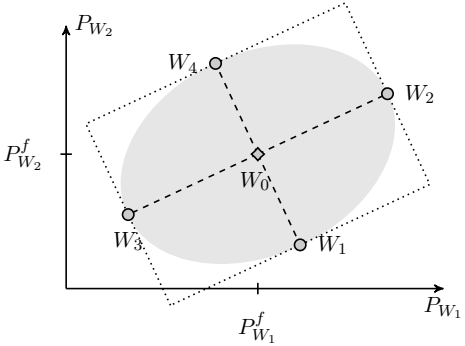


Analytical Reformulation for Gaussian Distributions



- We use a piecewise affine policy which interpolates system state between forecasted system state W_0 and end-point of the ellipsoid axes of the uncertainty set W_{1-4}.

Analytical Reformulation for Gaussian Distributions

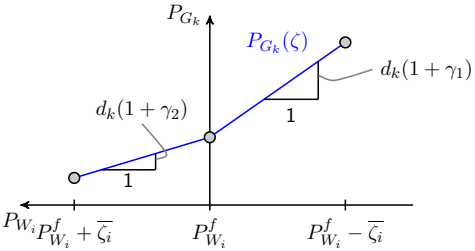


- Using results from chance-constrained optimization[4], an analytical reformulation of the linear chance constraints as SOC constraints can be applied.
- The semidefinite and SOC chance constraints are approximated.

[4] Arkadi Nemirovski. "On safe tractable approximations of chance constraints". In: *European Journal of Operational Research* 219.3 (2012), pp. 707–718

Corrective Control Policies

- To link forecasted system state W_0 with n_W matrix W_i, define generator participation factors d:



$$\Delta P_{G_k}(\zeta) = \Delta P_{W_i} d_k(1+\gamma_i)$$

γ_i accounts for non-linear change in losses

$\Rightarrow$ To obtain zero relaxation gap, loss penalty with weight μ in objective necessary :

$$\text{min. Gen. Cost } + \mu \sum_{i=1}^{n_W} \gamma_i$$

- Corrective control of generator voltage set-points (AVR), reactive power capabilities of wind farms (power factor 0.95 inductive to 0.95 capacitive)

Simulation Setup

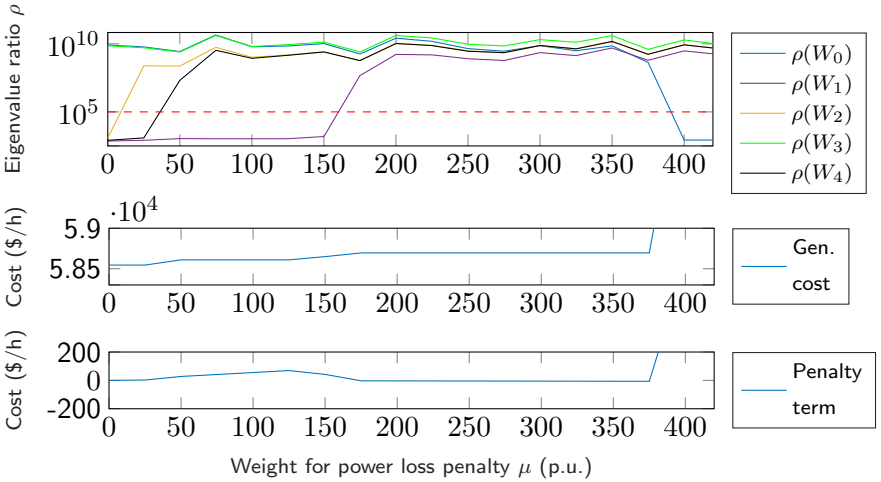
- Implemented in optimization toolbox JuMP/Julia and solver MOSEK V8
- Choose a violation limit of $\epsilon = 5\%$
- Heuristic for zero relaxation gap[5]: Ratio of 2nd to 3rd eigenvalue $\rho \geq 10^5$
- Near-global optimality guarantee[6]: $\delta_{opt} = \frac{f_0}{f_{\text{pen}}} \times 100\%$

[5] Daniel K Molzahn et al. "Implementation of a large-scale optimal power flow solver based on semidefinite programming". In: *IEEE Transactions on Power Systems* 28.4 (2013), pp. 3987–3998

[6] Ramtin Madani, Somayeh Sojoudi, and Javad Lavaei. "Convex relaxation for optimal power flow problem: Mesh networks". In: *IEEE Transactions on Power Systems* 30.1 (2015), pp. 199–211

Investigating Relaxation Gap

... for a IEEE 24 bus test system with 2 wind farms and rectangular uncertainty set.



$\Rightarrow$ Near-global optimality guarantee of 99.74%.

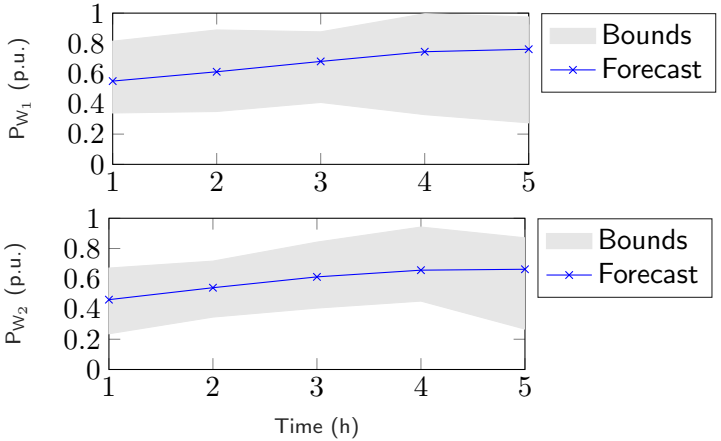
IEEE 118 bus test case

- Two wind farms with maximum rated power of 300 and 600 MW
- Using realistic forecast wind data from Danish wind farms
- Four designated generators compensate wind power deviations
- We compare our approaches to
  - AC-OPF without considering chance constraints
  - iterative chance constrained AC-OPF assuming Gaussian distributions from Schmidli et al.[7]
  - chance constrained DC-OPF for robust uncertainty set from Jabr[8]
- We use a Monte Carlo Analysis with 10'000 samples: Run MATPOWER AC power flows with control set-points from the respective approaches and evaluate empirical violation probability. Compare Cost of Uncertainty.

[7] Jeremias Schmidli et al. "Stochastic AC optimal power flow with approximate chance-constraints". In: *IEEE Power and Energy Society General Meeting*. Boston, US, 2016

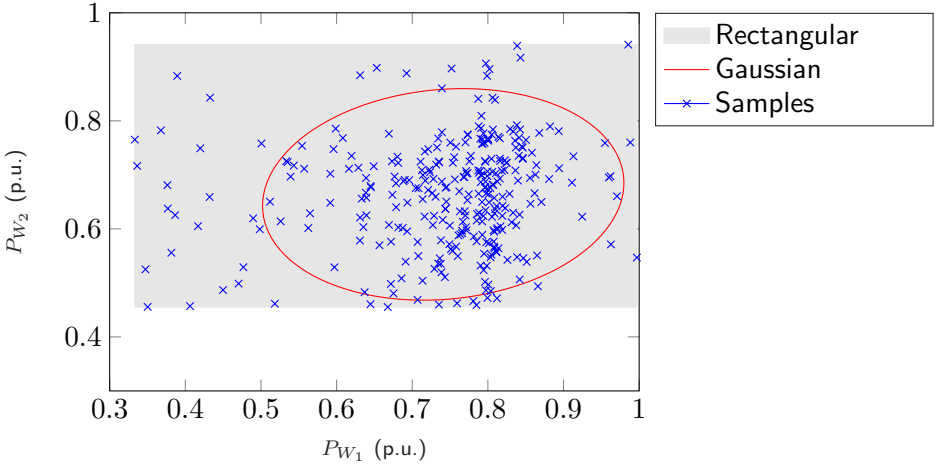
[8] Rabih A Jabr, Sami Karaki, and Joe Akl Korbane. "Robust multi-period OPF with storage and renewables". In: *IEEE Transactions on Power Systems* 30.5 (2015), pp. 2790–2799

Forecast Data



Construction of Uncertainty Sets

Comparison of robust and Gaussian uncertainty set obtained from the $N_s = 314$ scenarios sampled for hour 4:



Results - Monte Carlo Analysis

Empirical chance constraint violation probability using 10'000 samples and MATPOWER AC power flows (target 5%):

Time step (h)	1	2	3	4	5
Type of chance constraint: Active generator limit (%)					
AC-OPF w/o CC	**46.4**	**48.8**	**45.9**	**45.5**	**40.9**
CC-DC-OPF	**34.3**	**38.6**	**30.1**	**14.5**	2.0
CC-AC-OPF (Robust)	0.0	0.0	0.0	0.0	0.0
CC-AC-OPF (Gauss)	0.0	0.2	0.4	2.3	0.7
Iterative CC-AC-OPF	2.9	4.1	3.0	3.3	**5.7**

Results - Monte Carlo Analysis

Empirical chance constraint violation probability using 10'000 samples and MATPOWER AC power flows (target 5%):

Time step (h)	1	2	3	4	5
Type of chance constraint: Active power line limit (%)					
AC-OPF w/o CC	**17.7**	**18.8**	**14.9**	**32.5**	**46.5**
CC-DC-OPF	0.0	0.0	0.0	2.8	0.0
CC-AC-OPF (Robust)	0.0	0.0	0.0	0.0	0.0
CC-AC-OPF (Gauss)	4.6	3.7	0.9	2.6	**5.8**
Iterative CC-AC-OPF	1.6	2.0	3.6	4.2	**5.3**

Results - Monte Carlo Analysis

Empirical chance constraint violation probability using 10'000 samples and MATPOWER AC power flows (target 5%):

Time step (h)	1	2	3	4	5
Type of chance constraint: Bus voltages (%)					
AC-OPF w/o CC	0.0	0.1	0.2	1.0	4.3
CC-DC-OPF	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**
CC-AC-OPF (Robust)	0.0	0.0	0.0	0.0	0.0
CC-AC-OPF (Gauss)	0.7	1.9	4.3	**7.2**	**7.6**
Iterative CC-AC-OPF	0.0	0.0	0.0	0.0	0.0

Results - Cost of Uncertainty

Cost of chance-constrained approaches in relation to AC-OPF without considering uncertainty:

Time step (h)	1	2	3	4	5
Cost of Uncertainty (%)					
CC-AC-OPF (Rect) (%)	0.77	0.74	0.76	0.92	1.56
CC-AC-OPF (Gauss) (%)	0.52	0.46	0.47	0.49	0.51
Iterative CC-AC-OPF [9] (%)	0.52	0.47	0.47	0.49	0.52
CC-DC-OPF [3] (%)	-2.78	-2.83	-2.82	-2.67	-2.17

Results - Key findings for IEEE 118 bus test case

- Not considering chance constraints in the AC-OPF leads to violation of active line and generator limits.
- The DC-OPF formulation violates both voltage and active generator limits.
- Both AC-OPF formulations based on the assumption of Gaussian distribution do not achieve compliance with the chance constraints in the out-of-sample analysis for all considered time steps. In-sample chance constraints are fulfilled.
- The AC-OPF formulation based on robust optimization achieves compliance with chance constraints but at higher cost of uncertainty.
- We choose a penalty term of $\mu = 100$ and achieve a near-global optimality $> 99.99\%$ for the considered time steps.

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Considering N-1 Security Constraints

- N-1 security: An outage of any single component or line does not lead to a violation of system constraints.
- We focus on line outages, i.e. change in admittance matrix.

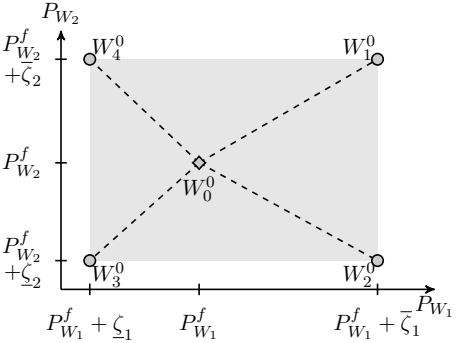
Source: Finanical Review

- We extend the formulation based on randomized and robust optimization to account for line outages.
- Similar to how the forecasted system state is linked to vertices of uncertainty set, we link forecasted intact system state to outaged states

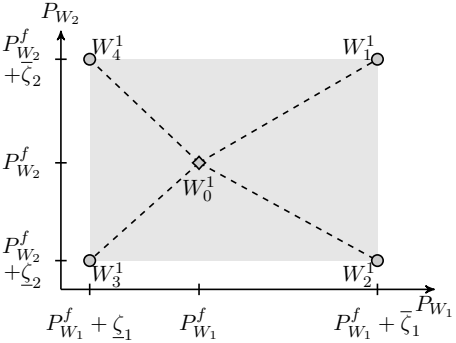
Randomized and Robust Optimization

... for the example of two wind farms and one considered outage:

for intact system state



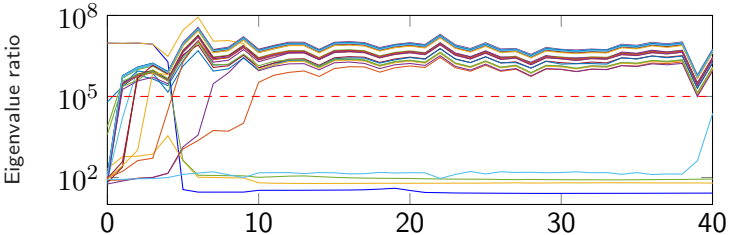
for outaged system state



- Instead of W_v we have W_v^c for each contingency c and vertex v!
- Challenge: obtain zero relaxation gap + scalability

Investigating Relaxation Gap

... for a IEEE 24 bus system with 3 wind farms and 2 outages, i.e. 27 W matrices:



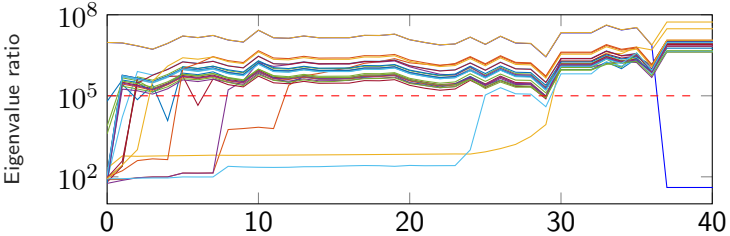
(a) – Number of iterations for uniform penalty term

Investigating Relaxation Gap

Systematic procedure to achieve zero relaxation gap:

1. Solve OPF with no penalty term. If we obtain rank-1 solutions for all included entries, we obtain zero relaxation gap and terminate.

2. Otherwise, we increase penalty weights for higher rank matrices W by a defined step-size $\Delta\mu$ and resolve OPF.
   $\Rightarrow$ Instead of $\mu \sum_{c,v} \gamma_v^c$ we use $\sum_{c,v} \mu_v^c \gamma_v^c$



(b) – Number of iterations for systematic penalty term

$\Rightarrow$ Near-global optimality guarantee is 99.77%.

Scalability

- We apply a chordal decomposition of the semidefinite constraint on matrix variables W. Enforce the semidefinite constraint only for the maximum cliques of graph describing power grid.

- Iterative solution algorithm:

  1. We initialize the algorithm by solving the OPF problem without chance and security constraints.
  2. MATPOWER AC power flows for each possible vertex and outage combination and evaluate the constraint violations. In case no violations occur, we terminate.
  3. Otherwise, we apply a constraint filtering: We identify the vertex and outage combinations which for at least one constraint dominate all other combinations in terms of magnitude of constraint violation[9]. Add dominating to OPF, solve OPF and return to 2).

[9] Florin Capitanescu et al. "Contingency filtering techniques for preventive security-constrained optimal power flow". In: *IEEE Transactions on Power Systems* 22.4 (2007), pp. 1690–1697

Outline

1. Motivation and Scope of Our Work
2. Convex Relaxations of Chance Constrained AC OPF
3. N-1 Security and Scalability
4. **Interconnected AC and HVDC Grids**
5. Conclusions

High Voltage Direct Current (HVDC) Grids

- Energinet and TenneT: Energy Island with 100 GW offhore wind in North Sea
  $\approx 13\%$ of US peak load
- Significant increase of HVDC in China: 24 projects in 2013
- China Southern Grid's Nan'ao island: 4 multi-terminal VSC HVDC grid with offshore wind
  $\Rightarrow$ first in the world!

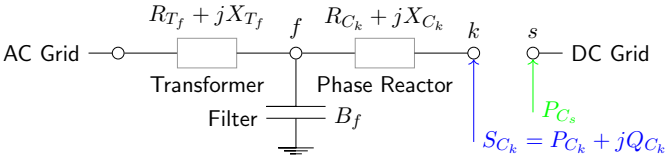
Source: Energinet

Source: Economist

Extension of AC-OPF

Semidefinite relaxation of chance constrained AC-OPF is extended to model interconnected AC and HVDC Grids based on Bahrami et al.[10]:

- DC grid is modeled as a purely resistive AC grid with generators operating at unity power factor
- VSC HVDC converter model with filter bus f, AC bus k and DC bus s:



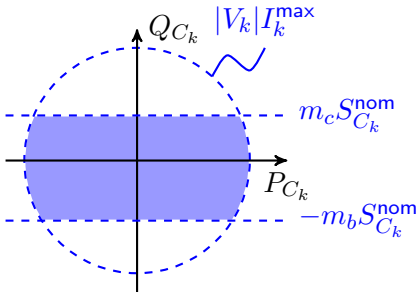
Power balance $\quad P_{C_k} + P_{C_s} + P^{\text{conv}}_{\text{loss},k} = 0$

Converter losses $\quad P^{\text{conv}}_{\text{loss},k} = a_k + c_k |I_k|^2$

[10] S. Bahrami et al. "Semidefinite Relaxation of Optimal Power Flow for AC-DC Grids". In: *IEEE Transactions on Power Systems* 32.1 (2017), pp. 289–304

HVDC Converter Control

- Independent control of active and reactive power (very fast!)
- PQ capability of HVDC converter:
  - Upper and lower limit on reactive power
  - Upper limit on apparent power



$\Rightarrow$ In our framework we can use the corrective control capabilities of the HVDC converter to balance wind power fluctuations and provide voltage support.

Contributions of our Work

- Using the combination of randomized and robust optimization we can state a tractable formulation of the chance constraints for interconnected AC and HVDC grids.
- We include corrective control of HVDC active and reactive power set-points.
- Compared to a DC-OPF formulation[11], using realistic forecast data and a Monte Carlo Analysis, we find our approach complies with all chance constraints whereas the DC-OPF formulation can lead to chance constraint violations.

[11] Roger Wiget, Maria Vrakopoulou, and Goran Andersson. "Probabilistic security constrained optimal power flow for a mixed HVAC and HVDC grid with stochastic infeed". In: *Power Systems Computation Conference (PSCC), 2014*. IEEE. 2014, pp. 1–7

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Conclusions & Outlook

- Tractable chance constrained AC-OPF formulation for both scenario-based and Gaussian uncertainty set which allows to
  - include the full AC-OPF formulation
  - corrective control policies
  - provide (near-)global optimality guarantees
- Applicability of the proposed framework to include N-1 security and interconnected AC and HVDC grids
- Future work is directed towards:
  - Distributionally Robust Formulations: Better trade-off between violation probability and cost of uncertainty
  - Decomposition techniques (Benders/ADMM)
  - Alternative methods to recover a feasible solution (joint work with Dan Molzahn, Argonne National Laboratory)

Thanks for Your Attention! Questions?

For more info:

- Venzke, Halilbasic, Markovic, Hug, and Chatzivasileiadis, Convex Relaxations of Chance Constrained AC Optimal Power Flow, in IEEE Transactions on Power Systems, vol. PP, no. 99, pp. 1-1. Available: `https://arxiv.org/pdf/1702.08372.pdf`
- Venzke and Chatzivasileiadis, Convex Relaxations of Security Constrained AC Optimal Power Flow under Uncertainty, accepted for presentation at Power Systems Computation Conference (PSCC), Dublin, 2018
- Venzke and Chatzivasileiadis, Convex Relaxations of Chance Constrained AC Optimal Power Flow for Interconnected AC and HVDC Grids, to be submitted, 2018