Probabilistic security-constrained optimal power flow including the controllability of HVDC lines

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Abstract— **In this paper we propose a security-constrained optimal power flow (SC-OPF) algorithm which incorporates the stochastic infeed of renewable sources, such as wind, as well as the controllability of HVDC lines. We extend our earlier work on SC-OPF problem and incorporate corrective control actions emanating form the controllability of the AGC lines. This is achieved by appropriately adjusting the HVDC setpoint after a component outage. The overall problem is formulated as chance constrained optimization program. We solve it using a scenario based methodology, which offers a-priori guarantees regarding the probability of constraint satisfaction. Moreover, we exploit recent developments in sampling based optimization allowing us to identify more optimal solutions in term of cost. To demonstrate the efficiency of our approach and the improvement in terms of cost due to the controllability offered by the HVDC lines, we apply to an IEEE benchmark network.**

I. Introduction

Integrating higher shares of renewable energy sources in power systems introduces additional uncertainties in their planning and operation. Higher transmission capacities and enhanced power system flexibility are required in order to maintain security and reliability in the system. New transmission technologies such as High Voltage Direct Current (HVDC) lines address both such needs. Still, tools are required which will take advantage of this additional power flow controllability and allow for new power system operating schemes. This paper proposes a security-constrained optimal power flow (SC-OPF) algorithm which incorporates the stochastic infeed of renewable sources, such as wind, as well as the controllability of HVDC lines. The algorithm seeks a solution which accomplishes two main objectives.

First, the system should be N-1 secure in a probabilistic sense. According to the ENTSO-E definition of N-1 criterion [1], a loss of an element within the TSO's responsibility area must not endanger the security of interconnected operation and lead to cascading outages. Here we consider systems with wind power generation but our approach can be easily extended to other type of renewable energy sources as well. In order to account for wind power forecast errors we should interpret security in a probabilistic sense. Toward this direction, [2], [3], [4], [5], concentrate on stochastic optimal power flow problems in a security constrained market clearing context. Nevertheless, no guarantees regarding the robustness of the resulting solution are provided, and only an a-posteriori analysis is conducted. In [6], a novel probabilistic framework with a-priori constraint satisfaction guarantees was proposed, resulting in a N-1 secure generation dispatch while accounting for wind uncertainty. The problem was formulated as a chance constrained optimization program and was solved using the so called scenario approach [7]. However, following this methodology provides only feasibility type probabilistic guarantees. In this paper, we exploit the result of [8] and use a sampling and discarding methodology to deal with chance constraints. This allows us to identify more optimal solutions in term of cost.

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Second, the algorithm should take advantage of the controllability offered by the HVDC lines. The focus here is on HVDC lines based on Voltage Source Converter technology (VSC-HVDC), which allow independent control of the active and reactive power flow. A typical OPF problem usually focusses only on the power injections of the generating units. The presence of HVDC lines in the system, however, introduces additional control variables. By actively changing their power flow, and due to their fast response, HVDC lines are able to undertake not only preventive but also corrective control actions. Thus, the optimization can not only determine the operating point in the base case, but can also compute the change in the HVDC power flow after a contingency occurs in order to maintain system security. In [9], such a security-constrained OPF is proposed, based on an AC-OPF formulation. It includes the post-contingency control of VSC-HVDC lines and demonstrates the benefits in terms of cost savings resulting from the ability of HVDC lines to act correctively. Motivated by this, we extend further the method presented in [6] by incorporating HVDC lines and their capability to offer post-disturbance control actions. The term disturbance here denotes not only component outages but also wind power forecast errors. The performance of our methodology is illustrated by means of case studies on the IEEE 30-bus network.

Section II provides a probabilistic SC-OPF formulation, including the controllability of the HVDC lines. In Section III we show how to deal with the chance constrained of the resulting optimization program, while Section IV provided a simulation study. Finally, Section V provides some concluding remarks and directions for future work.

II. Problem formulation

A. Power flow modeling

We consider a power network comprising of N_G generating units, N_L loads, N_l lines, N_b buses, N_w wind power generators, and N_{DC} HVDC lines. For the N-1 security analysis we take into account any single outage involving the tripping of a line, load, conventional generator and HVDC line. Denote then by I the set of indices corresponding to outages of all the components including also index "0" that corresponds to the base case of no outage.

Our formulation is based on a DC power flow model and on a post-disturbance operation of the system as presented in [6]. The disturbance could be thought of as the wind power forecast error and/or the outage of any component. Given a generation-load mismatch, the Automatic Generation Control (AGC) will drive the generation to a new operating point. Each generator adjusts its production compensating a percentage of the generation-load mismatch. In [6] the distribution vectors $d \in \mathbb{R}^{N_G}$ are introduced, and their elements denote the percentage with which each generating unit should change its production in response to the total mismatch occurred. The post-disturbance generation operating point is given by $P_G - dP_m$, where $P_m \in \mathbb{R}$ represents the generation-load mismatch.

The HVDC lines are included based on a simplified version of the modeling framework described in [9]. In [9] the VSC-HVDC lines were integrated in a Security-Constrained AC-OPF context. Equations have been included in the formulation, which coupled the VSC-HVDC variables such as the DC voltage V_{dc}, the AC converter voltage V_{ac}, the HVDC line losses based on the line resistance R_{dc}, and the modulation factor M with the AC power flow equations. Additionally, a VSC-HVDC capability curve in the form of an MVA circle has been incorporated. In this paper, as it is typical in a DC-OPF context, reactive power and dc line losses are neglected, while the voltage in the AC and DC side is always assumed equal to 1 p.u. We also consider no P-Q capability curve for the converters, but, similar to our assumptions for the AC lines, we assume that the dc line maximum power transfer is equal to the dc line active power limit.

In this paper, each HVDC line is approximated by two virtual voltage sources located at the two nodes where the HVDC line is connected. For each HVDC line, one additional variable is introduced representing the HVDC power flow. Let then $P_{DC} \in \mathbb{R}^{N_{DC}}$ represent the vector with the power flows on the HVDC lines. The balance between the active power injected in one node and the active power withdrawn at the other end of the line is ensured by assuming that the $P_{DC}^{in} = -P_{DC}^{out}$. The power flows of the rest of the lines are given by $P_l = AP_{inj}$, where $P_{inj} \in \mathbb{R}^{N_b}$ is the net power injection at the buses and A is a constant matrix that depends on the network admittances.

B. Preventive and corrective control

In a preventive SC-OPF we obtain setpoints for the controllable variables so that in the post-disturbance operating point where the system is driven as an effect of the automatic control actions (e.g. AGC), no operational limit violations occur. In our case the controllable variables are the generator dispatch and the power flow setpoint of the HVDC lines. Due to their fast response, however, the HVDC lines can react after the occurrence of a disturbance. This introduces a corrective security scheme, according to which HVDC lines can change their power flow setpoint depending on the event occurred. In the sequel we incorporate this scheme in a probabilistic SC-OPF framework.

C. Optimization problem

We aim to find a generation dispatch that minimizes the generation cost while satisfying the network constraints for the base case and all post-contingency operating points with a certain probability. We also consider a corrective control action for the HVDC lines by appropriately adjusting their setpoint for any outage.

Let $c_1, c_2 \in \mathbb{R}^{N_G}$ be generation cost vectors, and denote by $[c_2]$ a diagonal matrix with vector c_2 on the diagonal. The resulting optimization problem is given by

$$\min_{P_G, \{K^i\}_{i \in I}} c_1^T P_{G,t} + P_{G,t}^T [c_2] P_{G,t}, \tag{1}$$

subject to

$$\mathbf{1}_{1\times N_b}(C_G^0 P_G + C_w^0 P_w^f - C_L^0 P_L) = 0. \tag{2}$$

$$\mathbb{P}\Big(P_w \in \mathbb{R}^{N_w} |\ -\overline{P}_l^i \leq A^i P_{inj}^i(P_w) \leq \overline{P}_l^i \\ \underline{P}_G^i \leq P_G^i - d^i P_m^i(P_w) \leq \overline{P}_G^i \\ \underline{P}_{DC}^i \leq P_{DC}^i + K^i \leq \overline{P}_{DC}^i \\ \text{for all } i \in I \Big) \geq 1 - \varepsilon \tag{3}$$

$$\text{constraints inside (3) with } P_w = P_w^f \tag{4}$$

where

$$\begin{aligned} P_{inj}^i(P_w) &= C_G^i(P_G - d^i P_m^i(P_w)) \\ &\quad + C_w^i P_w - C_L^i P_L + C_{DC}^i(P_{DC} + K^i), \quad (5) \\ P_m^i(P_w) &= k_w^i(P_w - P_w^f) \\ &\quad - b_G^i P_G - b_w^i P_w + b_L^i P_L, \quad i \in I. \quad (6) \end{aligned}$$

Note that the probability $\mathbb{P}$ in (3) is meant with respect to the wind power P_w. We also emphasized in (3) the quantities that depend on the wind power P_w. Matrix C_G associates the production of each generator to the bus that is connected to. Matrices C_w, C_L, C_{DC} are defined analogously. Constants b_G^i, b_L^i, b_w^i, k_w^i are binary row vectors whose elements are either "0" or "1". A value of "1" corresponds to the tripped component for outage i. Note that the generation-load mismatch is equal to zero for the base case, $P_m^0(P_w^f) = 0$.

Constraint (2) represents the power balance equation for the deterministic case where the wind power is equal to its forecast P_w^f. The power balance is trivially satisfied for all other wind power instances under the assumption that sufficient reserves are procured to allow the generation-load mismatch to be compensated by the generator units according to the distribution vectors. The chance constraint (3) encodes that the fact that the inequalities therein should be satisfied with probability at least $1 - \varepsilon$. Equation (4) is the deterministic counterpart of (3), requiring all constraints inside the probability to be satisfied for the case where the wind power is equal to its forecast, i.e. $P_w = P_w^f$.

The first two inequalities inside the chance constraint, correspond to line and generation limits, respectively. The last constraint includes our proposed HVDC corrective control action. We assume that for any outage $i \in I$, the HVDC power flow setpoint can be adjusted accordingly. This adjustment is represented by a set of variables $K^i \in \mathbb{R}^{N_{DC}}$, $i \in I$, and the post-contingency setpoint is given by $P_{DC} + K^i$.

In the next section, we show how to solve it without introducing assumptions on the probability distribution of the uncertainty and while providing guarantees regarding the probability of constraint satisfaction. To facilitate our analysis, the chance constraint (3) can be written in a more compact notation as

$$\mathbb{P}\Big(\delta \in \mathbb{R}^{N_w} \mid Fx + f + g(\delta) \leq 0\Big) \geq 1 - \varepsilon, \tag{7}$$

where all matrices and vectors are of appropriate dimension. Note that x is a vector including the decision variables (P_G and K^i, $i \in I$) and $\delta \in \mathbb{R}^{N_w}$ plays the role of the uncertainty which for this case is the wind power.

III. Solving the chance constrained problem

A. The scenario approach

In [10] the authors introduce the so called scenario approach to address chance constrained optimization problems. According to this approach, the chance constraint is substituted with a finite number of hard constraints corresponding to different scenarios of the uncertainty vectors. Moreover, by using a sufficient number of scenarios it provides a-priori guarantees that the resulting solution satisfies the chance constraint with a certain confidence. Here, we follow an alternative scenario based methodology to deal with the chance constraint, which is proposed in [11] and results in a lower number of scenarios. This method includes two steps. In the first step, the scenario approach is used to determine, with a confidence of at least $1-\beta$, the minimum volume set that contains at least $1-\varepsilon$ probability mass of the uncertainty. Details on how to determine such a set can be found in [11], [12]. Here we denote this set by Δ. To compute this set, the number of scenarios we need to generate is given by

$$N \geq \frac{1}{\varepsilon}\frac{e}{e-1}\Big(\ln\frac{1}{\beta} + 2N_w - 1\Big). \tag{8}$$

In the second step, we use the probabilistically computed set Δ and formulate a robust problem where the uncertainty is confined in this set. The chance constraint (7) is substituted by the following robust constraint

$$Fx + f + Hg(\delta) \leq 0, \text{ for all } \delta \in \Delta \tag{9}$$

The interpretation of (9) is that the constraint should be satisfied for all values of $\delta \in \Delta$. Following [11], any feasible solution of this problem is feasible for the chance constraint (7) with a probability of at least $1-\beta$. To solve the resulting robust program the reader is referred to [13], [11].

The constraints in the probabilistic SC-OPF formulation, as well as in most of the optimization problems presented in this paper, exhibit a specific structure that can be exploited to achieve a computationally simpler problem. Specifically, the uncertainty appears only in the terms $Hg(\delta)$ that are additive in the constraint functions. Therefore, it suffices to calculate off-line the maximum values of the elements of $Hg(\delta)$ as δ varies within Δ and replace with it the term $Hg(\delta)$ in the constraints. Specifically, $Hg(\delta)$ is substituted with a vector whose elements are calculated as $\max_j H_k g(\delta^{(j)})$ with $j = 1, \ldots, 2^{N_w}$ denoting the indices of the vertices of the computed set Δ. Subscript k denotes the k-th row of matrix. The resulting problem is of the same size as each deterministic counterpart.

B. Sampling and discarding

The scenario approach described in the previous section provides feasibility type probabilistic guarantees but not optimality ones. To trade optimality, we use a subsequent development of the scenario approach, the so called sampling and discarding methodology [8], [14]. Here, we incorporate these results when using the scenario approach in the methodology of [11] where we seek to determine a set Δ which encloses the uncertainty with certain probability. Given N samples of the uncertainty, r of them (possibly corresponding to outliers) according to some rule and determine the set Δ is determined based only on the remaining samples $N-r$ samples. That way, the constructed Δ will have a smaller volume, thus reducing the conservatism of the solution of obtained via the procedure of Section III-A. Following [14], the number of scenarios that we need to generate in this case, so that we are able to eliminate r of them, is given by

$$N \geq \frac{2}{\varepsilon}\Big(\ln\frac{1}{\beta} + 2(N_x + r - 1)\Big) \tag{10}$$

Less conservative, but implicit bounds for the sample size (i.e. not solved explicitly with respect to N) are provided in [14], [8] and given ε, β and r, numerical inversion is required to compute N.

There are multiple ways to select which r samples to discard [14]; however, based on our simulation study the largest improvement in the cost is achieved when a greedy approach is adopted. We first solve the problem with N constraints and identify the ones that are active for the problem where we want to determine the minimum volume set Δ, or in other words the samples that lie on the facets of Δ. We then remove the one which results in the hyper-rectangle that leads to the highest reduction in the objective value $J(x)$. In case multiple samples on the same facet of Δ, we remove all of them at the same time. The reader is also referred to the next section, where the sampling and discarding procedure is illustrated by means of an example.

IV. Simulation results

The methodology developed in the previous sections is applied to the IEEE 30-bus network [15], which is modified to include two wind power generators (i.e. $N_w = 2$) at buses 7 and 19, and two VSC-HVDC lines connecting nodes 8 with 12, and 19 with 22. To generate wind power scenarios we used a Markov chain based model as described in [16], [12]. All optimization problems were solved using the solver CPLEX [17] via the MATLAB interface YALMIP [18].

Fig. 1 compares the operating costs with and without HVDC corrective control for three different cases. First, after executing an OPF, where no security criteria or forecast uncertainties are considered. Second, after running an SC-OPF, where the N-1 security criterion is included in the constraints. Third, a probabilistic SC-OPF (pSC-OPF) is run using the scenario approach of Section III-B, taking into account both security constraints and wind forecast uncertainties. In this case study, the forecasted wind power

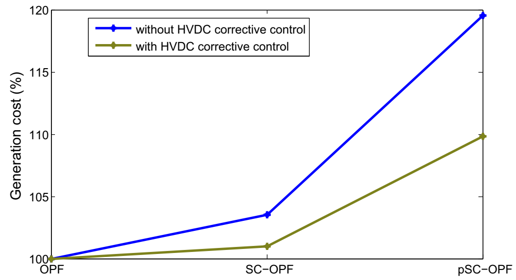

Fig. 1. Comparison of operating costs with and without HVDC corrective control. The OPF formulation represents the base-case dispatch where no security constraints and wind forecast errors are taken into account. SC-OPF results in a N-1 secure dispatch. pSC-OPF takes additionally into account the wind forecast uncertainties.

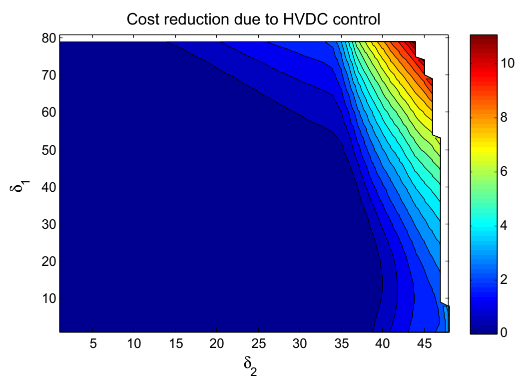

Fig. 2. Cost savings by enabling HVDC corrective control for different levels of wind penetration. The wind power outputs δ_1 and δ_2 are in MW. Cost savings are expressed in (%) and are calculated with respect to the same wind infeed but with no HVDC corrective control capability.

is assumed 79 MW at bus 7 and 40 MW at bus 19. The pSC-OPF results in higher operating costs than SCOPF or OPF, as it tries to identify a robust solution against wind forecast errors while keeping the system N-1 secure. As we can observe from Fig. 1, in both SCOPF and pSCOPF cases there is a decrease in the operating costs if we allow allowing the HVDC lines to react in case of contingency by offering corrective control actions. It is interesting to note though that the benefit in cost savings is five times higher (from 2% to 10%), if we take wind forecast uncertainty into account. This highlights the fact that operational flexibility – here, the HVDC controllability – can result in significant cost savings as higher shares of fluctuating generation with limited control, e.g. solar, wind, are integrated in the system.

In the second case study, we quantify the cost savings resulting from the corrective control capabilities of HVDC lines for different levels of wind penetration. Fig. 2 presents such cost savings for different combinations of wind generation. The wind generator at bus 7 (denoted by δ_1) varies its output between 0 and 80 MW, while the wind generator at bus 19 (denoted by δ_2) reaches an output of up to 50 MW. For power outputs above 80 MW at bus 7 or above 50 MW at bus 19, the problem becomes infeasible. As we can observe, the difference in costs, if the HVDC lines react or not after a contingency, is relatively small when the wind penetration is not very high. For this case study in particular, it seems that wind penetration below 60 MW at

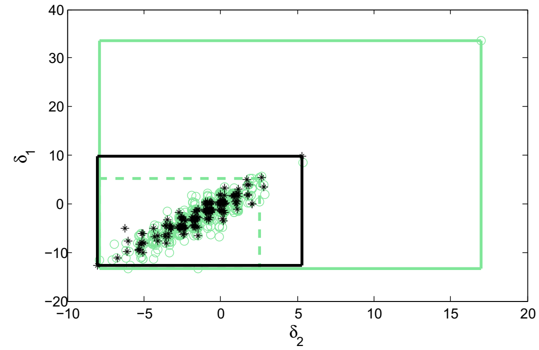

Fig. 3. Samples and rectangular sets Δ generated according to the methodology of Section III. The "black" and "green" color correspond to the scenario approach and the sampling and discarding approach, respectively. The outer rectangle (solid "green" line) corresponds to the case where we generate samples based on (10) but do not discard any of them, whereas the inner rectangle (dashed "green" line) corresponds to the situation where r out of N samples are discarded.

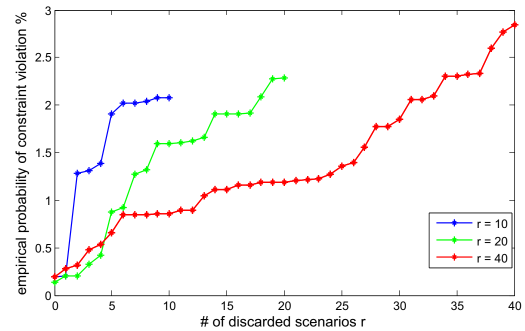

Fig. 4. Empirical probability of constraint violation calculated as the fraction out of 10,000 evaluation scenarios where the solution obtained by the optimization program violates at least one constraint is violated. Three different cases are simulated $r = 10, 20, 40$, and for each case the samples are discarded sequentially.

bus 7 and 40 MW at bus 17 results in cost savings of up to 2%. However, after a certain "tipping" point, the corrective control capabilities offered through the HVDC lines play a significant role, not only leading to cost savings up to 10%, but, in effect, allowing to integrate higher shares of wind generation.

For the rest of the section we show some features of the sampling and discarding approach of Section III-B and quantify the improvement afforded in terms of cost when this method is adopted. Fig. 3 shows the samples generated for the wind power of each generator. The pronounced triangular structure is due to fact that correlation was taken into account in the wind power model. When using the scenario approach of Section III-A we generate samples according to (8) and construct a set Δ. Since we have two wind generators, Δ is a rectangle as shown in Fig. 3 with "black". When employing the sampling and discarding procedure of Section III-B we generate samples according to (10). Since now we are alowed to discard r samples, the resulting rectangle (inner "green" rectangle) has smaller volume compared to what we would have if no samples were discarded (outer "green" rectangle). Since the volume of the inner rectangle is smaller than the "black" one, we expect the solution of the robust problem to

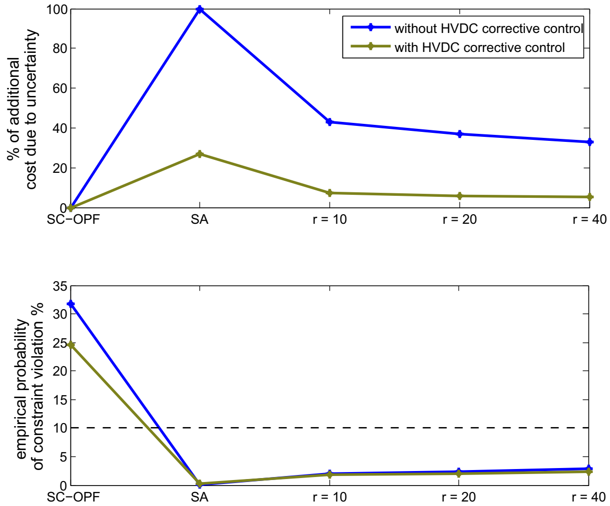

Fig. 5. Upper panel: Additional cost incurred due to a probabilistically robust design, relative to the cost due to a deterministic SC-OPF (for comparison its cost is set to zero). The second case corresponds to the situation where the scenario approach (SA) is adopted, whereas the last three correspond to problem instances where $r = 10, 20, 40$ samples are discarded, respectively. Lower panel: Empirical probability of constraint violation for the same problem instances with the upper panel.

be less conservative, leading to a lower cost.

To demonstrate the efficiency of our approach we carried out a Monte Carlo analysis. The solution of the optimization problem was evaluated against 10,000 wind power realizations different from those used in the optimization process. We then computed the empirical probability of constraint violation, calculated as the fraction out of the 10,000 scenarios where at least one of the constraints in (3) is violated. Note that the theoretical guarantees are given by ε, and in this case we set $\varepsilon = 10\%$. Fig. 4 shows for the cases $r = 10, 20, 40$ how the empirical probability of constraint violation changes as we progressively discard more samples. In coherence with the discussion of Section III-B, using a sampling and discarding approach, as the number of samples r we discard increases, the empirical probability of constraint violation tends to the theoretical ε-type guarantees.

This is also illustrated by means of Fig. 5. The upper panel of Fig. 5 shows the additional cost (as determined by the optimization problems) incurred due to a probabilistically robust design, relative to the cost due to a deterministic SC-OPF (for comparison its cost is set to zero). The second case corresponds to the situation where the scenario approach (SA) is adopted, whereas the last three correspond to problem instances where $r = 10, 20, 40$ samples are discarded, respectively. Notice that allowing for constraints to be discarded as in Section III-B, a significant reduction in the cost is achieved. Similarly, the lower panel of Fig. 5 shows the empirical probability of constraint violation. The more samples r we discard the closer this value gets to the theoretical value of ε ("dashed" line). Note that for the deterministic SC-OPF this value is significantly above ε, since no uncertainty is taken into account in the design phase.

V. Concluding remarks

In this paper we propose a probabilistic security-constrained optimization tool that takes advantage of the HVDC line controllability for post-disturbance control actions, and provides a-priori guarantees for the probability of constraint satisfaction. Such a tool could be useful for power system operators, providing additional flexibility during the N-1 security redispatching, as well as for transmission expansion planning studies, quantifying the benefit of HVDC controllability through the decrease in the expected generation costs. Current work concentrates on the optimal placement of the HVDC lines towards maximizing the wind power penetration, while minimizing the operational costs.

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