Remedial Actions to Enhance Stability of Low-Inertia Systems

Spyros Chatzivasileiadis, *Member, IEEE,* Thanh Long Vu, *Member, IEEE,* and Konstantin Turitsyn, *Member, IEEE*

***Abstract*—Increased penetration of renewable generation is expected to replace conventional generators and reduce system inertia. Future low-inertia systems are expected to include additional power sources to enhance stability by mimicking inertia and damping of conventional generators. This paper introduces such remedial actions in the formulation of direct methods for transient stability assessment. We extend our previous work on robust stability and resiliency certificates to include optimal tuning of inertia and damping coefficients for transient stability enhancement. The goal is to limit the fault-on trajectory in order to maintain the system inside its stability region. The advantage of this approach is the ability to guarantee system stability for a wider range of faults eliminating the need to carry out time-consuming simulations. An additional contribution of this paper is a novel formulation of the robust stability and resiliency certificates, which relaxes our optimization problem and allows to obtain significantly better results.**

***Index Terms*—Lyapunov functions, Transient Stability, virtual inertia,**

I. Introduction

Increased environmental awareness and technological advancements lead to higher shares of renewable energy sources and other power-electronic connected generators. Replacing old conventional generators with these new generating sources results to systems with lower rotating inertia. It is well known that in the occurence of a fault, low-inertia systems are more prone to instability, as they can move in less time outside the stability region of the system [1]. To mitigate this effect, several approaches have been proposed, taking advantage e.g., of the wind turbine rotational inertia [2] or through additional power sources, e.g., [3], [4]. Industrial approaches for inertia mimicking by incorporating storage and appropriate control loops in inverter-connected generators already exist in the literature [5]. Future power systems will most probably include additional power sources connected close to renewable generators with the task to assist in maintaining the frequency and rotor angle stability of the system during faults.

Before moving on with the goal of this paper, four comments concerning the use of these additional power sources are in order. First, given that faults in power systems are relatively rare events and the transient phase of the fault duration is usually in the range of seconds, these power sources are not expected to only be used for maintaining the transient stability of the system. Instead, they will probably serve multiple purposes, such as real time balancing of the uncertain RES infeed, electric power arbitrage to benefit from price differences at different time periods, etc. Nevertheless, similar to the up and down regulating reserves, a portion of their capacity can be reserved so that they can assist in power system stability in the occurence of a fault. Second, if the renewable generators are equipped with the appropriate controllers, similar functions can be carried out by themselves, as long as they are able to reserve a guaranteed amount of power, see e.g [2]. Third, given the flexibility provided by an additional power source, we can modulate the injected power to not only mimic the inertia of a conventional generator, but also increase the damping coefficient. In this paper, we consider changes both in the inertia and the damping coefficient to guarantee power system stability in the event of a fault. This approach is also followed in several papers, e.g. [2], [6], [3]. Fourth, the methods presented in this paper are agnostic of the type of the additional power source that will be used to provide the tuned values of inertia and damping. In the rest of this paper, by tuning of inertia and damping of low-inertia generators, we mean the appropriate tuning of the additional power sources associated with these generators.

The goal of this paper is to extract robust certificates to guarantee the transient stability of the system in a certain region, avoiding time consuming time-domain simulations. In that, we also consider inertia and damping coefficient tuning as remedial actions for low-inertia systems. Similar to direct energy methods [7], the goal of such certificates is to determine an as large as possible region of attraction, which will cover a large set of the most common power system operating conditions and faults. This reduces the need for time-domain simulations for transient stability assessment to only a small set of severe faults. This paper extends the work previously presented in [8] and [9]. Ref. [8] generalizes the idea of energy methods, and extends the concept of energy function to a more general Lyapunov Functions Family (LFF) constructed via Semidefinite-Programming techniques. Ref. [9] extends this method by introducing a robust stability and a robust resiliency certificate, guaranteeing system stability for a set of operating points or a set of faults respectively. Ref. [9] also introduced quadratic Lyapunov certificates, through which the stability region can be found in polynomial time.

This paper has two main contributions. First, it incorporates remedial actions in direct methods for transient stability assessment. It expands the methods introduced in [9] by incorporating the tuning of the inertia and damping coefficients. The goal is to appropriately tune inertia and damping during a fault in order to keep the fault trajectory inside the region of attraction of our system. By doing that, we can extend the range of faults for which we can provide guarantees that the system will sustain. Second, it introduces a rescaling factor Λ in the formulations for the robust stability and resiliency certificates, that relaxes the problem and allows us to obtain better results.

This paper is structured as follows. Sections II and III

Spyros Chatzivasileiadis, Thanh Long Vu, and Konstantin Turitsyn are with the Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA, 02139 USA, e-mail: {chatziva, longvu, turitsyn}@mit.edu.

present our modeling approach and outline the formulation for the robust stability certificate. Section IV introduces the rescaling parameter Λ. Section V describes the algorithm to determine the minimum values for the inertia and damping coefficients to enhance the transient stability of the system. Finally, Section VI presents a numerical example to demonstrate the algorithm. Section VII discusses the approach and Section VIII concludes.

II. Network Model and Emergency Control Problem

A. Network Model

Consider a power transmission grid including conventional generators, renewable generators, loads, and transmission lines connecting them. In this paper we consider the standard structure-preserving model to describe components and dynamics in power systems [10]. This model naturally incorporates the dynamics of generators' rotor angle as well as the response of load power output to frequency deviation. Although it does not model the dynamics of voltages in the system, in comparison to the classical swing equation with constant impedance loads, the structure of power grids is preserved in this model.

Mathematically, the grid is described by an undirected graph $\mathcal{G}(\mathcal{N},\mathcal{E})$, where $\mathcal{N}=\{1,2,\dots,|\mathcal{N}|\}$ is the set of buses and $\mathcal{E}\subseteq\mathcal{N}\times\mathcal{N}$ is the set of transmission lines connecting those buses. Here, $|A|$ denotes the number of elements in the set A. The sets of generator buses and load buses are denoted by $\mathcal{G}$ and $\mathcal{L}$ and labeled as $\{1,...,|\mathcal{G}|\}$ and $\{|\mathcal{G}|+1,...,|\mathcal{N}|\}$. We assume that the grid is lossless with constant voltage magnitudes $V_k, k\in\mathcal{N}$, and the reactive powers are ignored.

By power balancing we obtain the structure-preserving model of power systems as:

$$m_k\ddot{\delta_k}+d_k\dot{\delta_k}+\sum_{j\in\mathcal{N}_k}a_{kj}\sin(\delta_k-\delta_j)=P_{m_k}, k\in\mathcal{G}, \quad (1a)$$

$$d_k\dot{\delta_k}+\sum_{j\in\mathcal{N}_k}a_{kj}\sin(\delta_k-\delta_j)=-P^0_{d_k}, k\in\mathcal{L}, \quad (1b)$$

where $a_{kj}=V_kV_jB_{kj}$, m_k are generators' moments of inertia, and d_k are the generators' damping coefficient and the loads' frequency constant. The equations (1a) represent the dynamics at generator buses and the equations (1b) the dynamics at load buses. For a more detailed derivation of (1), the reader can refer to [9], [10].

The system described by equations (1) has many stationary points with at least one stable corresponding to the desired operating point. Mathematically, the state of (1) is presented by $\delta=[\delta_1,...,\delta_{|\mathcal{G}|},\dot{\delta}_1,...,\dot{\delta}_{|\mathcal{G}|},\delta_{|\mathcal{G}|+1},...,\delta_{|\mathcal{N}|}]^T$, and the desired operating point is characterized by the buses' angles $\delta^*=[\delta^*_1,...,\delta^*_{|\mathcal{G}|},0,\dots,0,\delta^*_{|\mathcal{G}|+1},...,\delta^*_{|\mathcal{N}|}]^T$. This point is not unique since any shift in the buses' angles is also an equilibrium. However, it is unambiguously characterized by the angle differences $\delta^*_{kj}=\delta^*_k-\delta^*_j$ that solve the following system of power-flow like equations:

$$\sum_{j\in\mathcal{N}_k}a_{kj}\sin(\delta^*_{kj})=P_k, k\in\mathcal{N}, \quad (2)$$

where $P_k=P_{m_k}, k\in\mathcal{G}$, and $P_k=-P^0_{d_k}, k\in\mathcal{L}$.

Assumption 1: There is a solution δ^* of equations (2) such that $|\delta^*_{kj}|\leq\gamma<\pi/2$ for all the transmission lines $\{k,j\}\in\mathcal{E}$.

We recall that for almost all power systems this assumption holds true if we have the following synchronization condition, which is established in [11],

$$||L^\dagger p||_{\mathcal{E},\infty}\leq\sin\gamma. \quad (3)$$

Here, $L^\dagger$ is the pseudoinverse of the network Laplacian matrix, $p=[P_1,...,P_{|\mathcal{N}|}]^T$, and $||x||_{\mathcal{E},\infty}=\max_{\{i,j\}\in\mathcal{E}}|x(i)-x(j)|$. In the sequel, we denote as $\Delta(\gamma)$ the set of equilibrium points δ^* satisfying that $|\delta^*_{kj}|\leq\gamma<\pi/2,\forall\{k,j\}\in\mathcal{E}$. Then, any equilibrium point in this set is a stable operating point [11].

B. Emergency Control Problem

In normal conditions, a power grid operates at a stable equilibrium point of the pre-fault dynamics. After the initial disturbance (in this paper we consider line tripping) the system evolves according to the fault-on dynamics laws and moves away from the pre-fault equilibrium point δ^*_{pre}. At the clearing time $\tau_{clearing}$, the fault is cleared, the system is at the fault-cleared state $\delta_0=\delta_F(\tau_{clearing})$, and then the tripped line is reclosed. Hence, the system configuration is the same as the pre-fault one and the power system experiences the post-fault transient dynamics. The transient stability of the post-fault dynamics is certified if the system converges from the fault-cleared state to the post-fault stable equilibrium point δ^*_{post}, or more clearly, if the fault-cleared state stays inside the region of attraction of the post-fault stable equilibrium point.

In this paper, we assume that when a line tripping occurs, the system operator can immediately send signals to simultaneously adjust the inertia and damping of the low-inertia generators without any communication and regulation delays (see Section VII for a short discussion about these delays). We also assume that the tuned values of inertia and damping can be kept for at least a time period $[0,\tau_{clearing}]$. Here it should be noted that, since $\tau_{clearing}$ is in the range of hundreds of milliseconds, one of the advantages of the approach presented in this paper is that it requires a very limited amount of power and energy from the additional power sources. Our emergency control problem is how to appropriately tune the inertia and damping of the low-inertia generators (with the help of an additional power sources) to compensate for the disturbance such that after the given clearing time $\tau_{clearing}$, the fault-cleared state is still inside the region of attraction of the post-fault stable equilibrium point δ^*_{post}.

If this objective can be obtained, then at the clearing time $\tau_{clearing}$, the fault is cleared, the inertia and damping of the low-inertia generators are brought back to their initial values, and the power system will evolve according to the post-fault dynamics from the fault-cleared state to the stable post-fault equilibrium point.

III. Quadratic Lyapunov Function-based Transient Stability Certificate

In this section, we recall our recently introduced quadratic Lyapunov function-based transient stability certificate for power systems in [9]. To this end, we separate the nonlinear couplings and the linear terminal system in (1). For brevity, we denote the stable post-fault equilibrium point for which we want to certify stability as δ^*. Consider the state vector $x = [x_1, x_2, x_3]^T$, which is composed of the vector of generator's angle deviations from equilibrium $x_1 = [\delta_1 - \delta_1^*, \ldots, \delta_{|\mathcal{G}|} - \delta_{|\mathcal{G}|}^*]^T$, their angular velocities $x_2 = [\dot{\delta}_1, \ldots, \dot{\delta}_{|\mathcal{G}|}]^T$, and vector of load buses' angle deviation from equilibrium $x_3 = [\delta_{|\mathcal{G}|+1} - \delta_{|\mathcal{G}|+1}^*, \ldots, \delta_{|\mathcal{N}|} - \delta_{|\mathcal{N}|}^*]^T$. Let E be the incidence matrix of the graph $\mathcal{G}(\mathcal{N}, \mathcal{E})$, so that $E[\delta_1, \ldots, \delta_{|\mathcal{N}|}]^T = [(\delta_k - \delta_j)_{\{k,j\} \in \mathcal{E}}]^T$. Let the matrix C be $E[I_{m\times m}\ O_{m\times n}; O_{(n-m)\times 2m}\ I_{(n-m)\times(n-m)}]$. Then

$$Cx = E[\delta_1 - \delta_1^*, \ldots, \delta_{|\mathcal{N}|} - \delta_{|\mathcal{N}|}^*]^T = [(\delta_{kj} - \delta_{kj}^*)_{\{k,j\}\in\mathcal{E}}]^T.$$

Consider the vector of nonlinear interactions F in the simple trigonometric form: $F(Cx) = [(\sin\delta_{kj} - \sin\delta_{kj}^*)_{\{k,j\}\in\mathcal{E}}]^T$. Denote the matrices of moment of inertia, frequency controller action on governor, and frequency coefficient of load as $M_1 = \mathrm{diag}(m_1, \ldots, m_{|\mathcal{G}|}), D_1 = \mathrm{diag}(d_1, \ldots, d_{|\mathcal{G}|})$ and $M = \mathrm{diag}(m_1, \ldots, m_{|\mathcal{G}|}, d_{|\mathcal{G}|+1}, \ldots, d_{|\mathcal{N}|})$. In state space representation, the power system (1) can be then expressed in the following compact form:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= M_1^{-1} D_1 x_2 - S_1 M^{-1} E^T S F(Cx) \\ \dot{x}_3 &= -S_2 M^{-1} E^T S F(Cx) \end{aligned} \quad (4)$$

where $S = \mathrm{diag}(a_{kj})_{\{k,j\}\in\mathcal{E}}, S_1 = [I_{m\times m} \quad O_{m\times n-m}], S_2 = [O_{n-m\times m} \quad I_{n-m\times n-m}], n = |\mathcal{N}|, m = |\mathcal{G}|$. Equivalently, we have

$$\dot{x} = Ax - BF(Cx), \quad (5)$$

with the matrices A, B given by the following expression:

$$A = \begin{bmatrix} O_{m\times m} & I_{m\times m} & O_{m\times n-m} \\ O_{m\times m} & -M_1^{-1} D_1 & O_{m\times n-m} \\ O_{n-m\times m} & O_{n-m\times m} & O_{n-m\times n-m} \end{bmatrix},$$
$$B = \begin{bmatrix} O_{m\times|\mathcal{E}|}; & S_1 M^{-1} E^T S; & S_2 M^{-1} E^T S \end{bmatrix}.$$

The construction of quadratic Lyapunov function is based on the bounding of the nonlinear term F by linear functions of the angular differences. Particularly, we observe that for all values of $\delta_{kj} = \delta_k - \delta_j$ staying inside the polytope $\mathcal{P}$ defined by the inequalities $|\delta_{kj}| \leq \pi/2$, we have:

$$g_{kj}(\delta_{kj} - \delta_{kj}^*)^2 \leq (\delta_{kj} - \delta_{kj}^*)(\sin\delta_{kj} - \sin\delta_{kj}^*) \leq (\delta_{kj} - \delta_{kj}^*)^2 \quad (6)$$

where

$$g_{kj} = \min\{\frac{1 - \sin\delta_{kj}^*}{\pi/2 - \delta_{kj}^*}, \frac{1 + \sin\delta_{kj}^*}{\pi/2 + \delta_{kj}^*}\} = \frac{1 - \sin|\delta_{kj}^*|}{\pi/2 - |\delta_{kj}^*|} \quad (7)$$

Let $g = \min_{\{k,j\}\in\mathcal{E}} g_{kj}$. For each transmission line $\{k,j\}$ connecting generator buses k and j, define the corresponding flow-in boundary segment $\partial\mathcal{P}_{kj}^{in}$ of the polytope $\mathcal{P}$ by equations/inequalities $|\delta_{kj}| = \pi/2$ and $\delta_{kj}\dot{\delta}_{kj} < 0$, and the flow-out boundary segment $\partial\mathcal{P}_{kj}^{out}$ by $|\delta_{kj}| = \pi/2$ and $\delta_{kj}\dot{\delta}_{kj} \geq 0$. Consider the quadratic Lyapunov function $V(x) = x^T P x$ and define the following minimum value of the Lyapunov function $V(x)$ over the flow-out boundary $\partial\mathcal{P}^{out}$ as:

$$V_{\min} = \min_{x\in\partial\mathcal{P}^{out}} V(x), \quad (8)$$

where $\partial\mathcal{P}^{out}$ is the union of $\partial\mathcal{P}_{kj}^{out}$ over all the transmission lines $\{k,j\} \in \mathcal{E}$ connecting generator buses. We have the following result, which is a corollary of Theorem 1 in [9]. Hence, the proof is omitted.

Theorem 1: (Transient Stability Certificate) *Consider a power system with the post-fault equilibrium point $\delta^* \in \Delta(\gamma)$ and the fault-cleared state x_0 staying in the polytope $\mathcal{P}$. Assume that there exists a positive definite matrix P such that*

$$\begin{bmatrix} \bar{A}^T P + P\bar{A} + \frac{(1-g)^2}{4} C^T C & PB \\ B^T P & -I \end{bmatrix} \leq 0 \quad (9)$$

and

$$V(x_0) < V_{\min} \quad (10)$$

where $\bar{A} = A - \frac{1}{2}(1+g)BC$. Then, the system trajectory of (1) *will converge from the fault-cleared state x_0 to the stable equilibrium point δ^*.*

Therefore, a sufficient condition for the transient stability of the post-fault dynamics is the existence of a positive definite matrix P satisfying the LMI (9) and the Lyapunov function at the fault-cleared state is smaller than the critical value $V_{\min}$ defined as in (8). We will utilize this condition to design the emergency control in the next section.

IV. Introducing the rescaling parameter Λ

In this paper we introduce a parameter Λ with which we can rescale matrices B and F, see (11).

$$\dot{x} = Ax - B\Lambda^{-1}\Lambda F(Cx) \quad (11)$$

This additional degree of freedom relaxes condition (9) and makes the search for an appropriate matrix P easier.

As long as Λ is a diagonal matrix, from (6) we get:

$$g\Lambda(\delta_{kj} - \delta_{kj})^2 \leq (\delta_{kj} - \delta_{kj})\Lambda(sin\delta_{kj} - sin\delta_{kj}^*) \leq \Lambda(\delta_{kj} - \delta_{kj})^2,$$

and hence

$$(\Lambda F(Cx) - g\Lambda Cx)^T(\Lambda F(Cx) - \Lambda Cx) \leq 0, \quad \forall x \in \mathcal{P}.$$

Following the same procedure as in [9], we derive the condition (proof is omitted due to space limitations):

$$\begin{bmatrix} \bar{A}^T P + P\bar{A} + \frac{(1-g)^2}{4} C^T \Lambda^T \Lambda C & PB \\ B^T P & -\Lambda^T\Lambda \end{bmatrix} \leq 0 \quad (12)$$

where $\bar{A} = A - \frac{1}{2}(1+g)BC$.

By setting $Q = \Lambda^T\Lambda$, with $Q > 0$ and diagonal, note that (12) is equivalent to (9) for any choice of Q. We can therefore let our solver freely determine an appropriate matrix Q to find a suitable P.

A. *Rescaling parameter Λ in the fault case*

Equation (13) represents the system dynamics when a line tripping has occured, with the matrix D removing the corresponding tripped line from the matrix $BF(Cx)$.

$$\dot{x} = Ax - B\Lambda^{-1}\Lambda F(Cx) + B\Lambda^{-1} D_{\{u,v\}}\Lambda \sin\delta_{F_{uv}} \quad (13)$$

Following the same derivations as in [9], and since Λ and $D_{\{u,v\}}$ are diagonal matrices, the necessary condition is (proof omitted due to space limitations):

$$\begin{bmatrix} \bar{A}^T P + P\bar{A} + \frac{(1-g)^2}{4} C^T \Lambda^2 C & PB \\ B^T P & -(\Lambda^{-2} + \mu D^2)^{-1} \end{bmatrix} \leq 0 \quad (14)$$

where $\bar{A} = A - \frac{1}{2}(1+g)BC$. Similar to the transient resiliency certificate in [9], the system is guaranteed to remain in the stability region as long as the critical clearing time is $\tau_{clearing} \leq \mu V_{\min}$. (proof omitted due to space limitations)

Equation (14) is nonlinear as it includes both Λ^2 and Λ^{-2}. However, matrix D is a diagonal sparse matrix. Considering a single fault, D has only a single non-zero value in the diagonal, corresponding to the line to be removed. As a result, we can remove the non-linearity by fixing this element in Λ. Assume for example a 3-bus system with three lines, where we study the tripping of the first line. Then the D matrix will be $diag([1\ 0\ 0])$. Assuming that $\Lambda = diag([\lambda_1\ \lambda_2\ \lambda_3])$ and $Q = diag([q_1\ q_2\ q_3]) = \Lambda^T\Lambda$, then $(\Lambda^{-2} + \mu D^2)^{-1} = diag([\frac{\lambda_1^2}{1+\mu\lambda_1^2}\ \lambda_2^2\ \lambda_3^2]) = diag([\alpha\ q_2\ q_3])$, where $q_1 = \frac{\alpha}{1-\alpha\mu}$ is a constant parameter.

V. Algorithm

Assume a power system described by the inertia and damping coefficients m_0, d_0. Assume the Lyapunov function $V_0(x) = x^T P_0 x$ for this system, with P_0 derived through (12), and its corresponding region of attraction $V_{\min} = \min_{x \in \partial \mathcal{P}^{out}} V(x)$. Our goal is to determine inertia and damping coefficients m_F^*, d_F^* during the fault-on dynamics x_F so that the fault-cleared state $x_0 = x_F(\tau_{clearing})$ remains inside the region of attraction of our initial system, i.e. $V(x_0) < V_{min}$.

To achieve this, our algorithm follows the following steps:

1) Find a positive definite matrix P_0 for the initial system m_0, d_0 satisfying the LMI (12).
2) Determine $V_{\min} = \min_{x \in \partial \mathcal{P}^{out}} x^T P_0 x$.
3) Determine the required critical clearing time $\tau_{clearing}$ and set $\mu_F = \frac{\tau_{clearing}}{V_{\min}}$
4) For given μ_F, vary $q_F, m_F \leq \overline{m}_F, d_F \leq \overline{d}_F$ and for each fixed value of inertia and damping, find a positive definite matrix P_F so that (here q_F is the fixed parameter in (14)):
   a) Condition (14) is fulfilled, *and*
   b) P_F satisfies constraint $P_0 \succeq P_F$.
5) If P_F is found:
   a) Fix P_F, and minimize over d_F s.t. condition (14)
   b) Find new $P_F(d_F^*)$
   c) Fix $P_F(d_F^*)$ and minimize over m_F s.t. (14)
   d) Find new $P_F(m_F^*, d_F^*)$
   e) Set $\overline{m}_F = m_F^*,\ \overline{d}_F = d_F^*$ and go to Step 3 or 4
6) If P_F is not found, go to Step 4.
7) After k iterations, reset $\overline{m}_F, \overline{d}_F$ to the initial upper bounds and go to Step 1, finding a new matrix P_0.

As it is obvious, the algorithm can find several sets of values m_F^*, d_F^* for which these conditions will hold. By selecting the smallest of these values, we can tune our power sources and keep these values during the time period $[0, \tau_{clearing}]$. At the clearing time $\tau_{clearing}$ the fault is cleared and the inertia and damping can be tuned back to their initial values.

By satisfying (14), we can prove that $x_0^T P_F x_0 < V_{\min}$, similar to Theorem 3 in [9] (proof omitted due to space limitations). Together with the condition $P_0 \succeq P_F$, this leads to $x_0^T P_0 x_0 \leq x_0^T P_F x_0 < V_{\min}$. Applying Theorem 1, we conclude that the fault-cleared state stays inside the region of attraction and therefore the post-fault dynamics are stable.

VI. Numerical Example

To illustrate the algorithm of this paper, we consider the simple yet non-trivial system of three generators. Future work will demonstrate this approach to larger systems. Assuming a high penetration of renewable energy sources, all generators are low-inertia systems each integrated with an additional power source to allow tuning of inertia and damping.

The susceptance of the transmission lines are assumed at $B_{12} = 0.739$ p.u., $B_{13} = 0.5$ p.u., and $B_{23} = 0.5$ p.u. The inertia and damping of all generators at the normal operating condition are $m_k = 0.5$ p.u., $d_k = 1$ p.u. Assume that the line between generators 1 and 2 is tripped. During the fault, the time-invariant terminal voltages are $V_1 = 1.0566$ p.u., $V_2 = 1.0502$ p.u., $V_3 = 1.0170$ p.u. and mechanical power injection/withdrawal per bus is $P_1 = -1$ p.u., $P_2 = 0.9$ p.u., $P_3 = 0.1$ p.u.. The pre-fault and post-fault equilibrium point is calculated from (2): $\delta^* = [-0.5127\ 0.4939\ 0.0957\ 0\ 0\ 0]^T$. Hence, the equilibrium point stays in the polytope defined by the inequality $|\delta_{kj}| < 1.1$ rad. As such, we can take $g = (1 - \sin(1.1))/(\pi/2 - 1.1)$. Using CVX in MATLAB to solve the LMI (9), we can obtain the Lyapunov function $V(x) = x^T P_0 x$ where

$$P_0 = \begin{bmatrix} 18.49 & -0.96 & -1.66 & 5.47 & 1.54 & 0.92 \\ -0.96 & 18.49 & -1.66 & 1.54 & 5.47 & 0.92 \\ -1.66 & -1.66 & 19.20 & 0.92 & 0.91 & 6.10 \\ 5.47 & 1.54 & 0.92 & 7.01 & 0.66 & -0.17 \\ 1.54 & 5.47 & 0.91 & 0.66 & 7.02 & -0.17 \\ 0.92 & 0.92 & 6.10 & -0.17 & -0.17 & 7.86 \end{bmatrix}$$

The minimum value $V_{\min}$ is $V_{\min} = 2.7087$. We set the critical fault clearing time to $\tau_{clearing} = 200ms$ and thus, $\mu = \tau_{clearing}/V_{\min} = 0.073837$.

Executing the algorithm described in Section V, we find $m_F^* = [0.53726\ \ 0.57554\ \ 0.42882]$, $d_F^* = [2.402\ 2.0866\ 1.0779]$. To verify that the system maintains its transient stability, we ran time-domain simulations, changing the inertia and damping values to the tuned ones during the fault-on dynamics. Fig. 1 presents the angle and frequency deviations during the tripping of line 1-2. Fig. 2 presents how the growth rate of the Lyapunov function is limited during the fault-on dynamics by tuning inertia and damping.

There are two additional points we wish to discuss here. The algorithm described in Section V can also be executed using the LMI conditions without the rescaling factor Λ, i.e. condition (9), and condition (13) with $\Lambda = I$. In that case we obtain $m_F^{*'} = [0.69842\ \ 1.22173\ \ 2.2866]$, $d_F^{*'} = [2.6621\ 2.436\ 1.56573]$. It is obvious that with the help of the rescaling factor Λ we can obtain much lower values for inertia and damping. Second, some remarks about the obtained results follow. We observe that the minimum inertia coefficient for the third generator is lower that its nominal value, i.e.

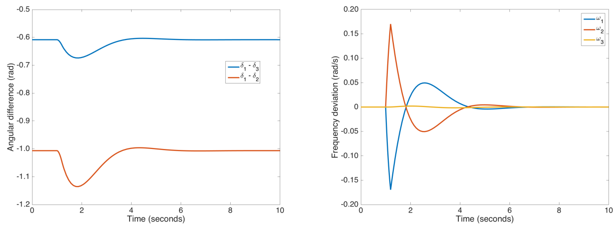

Fig. 1: Angular differences and generator frequencies after damping and inertia control with $\tau_{clearing} = 200$ ms.

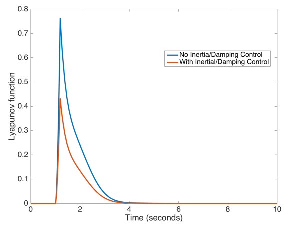

Fig. 2: Comparison of the quadratic Lyapunov function $V(x) = x^T P x = (\delta - \delta^*)^T P(\delta - \delta^*)$ before and after the damping and inertia control.

$m_F^*(3) = 0.42882 < 0.5$. Assuming that our power source is a type of storage, we see that it might be given the opportunity to store power from the grid and actually be remunerated for the service it offers. An additional remark here is that the damping coefficients seem to play a more important role than the inertia coefficients, as their increase is more significant. Finally, we observe that the major increase is on the generators 1 and 2 which are adjacent to the tripped line 1-2, while for the generator 3, the change in both is almost insignificant small.

VII. Discussion

In this paper we directly tuned the damping and inertia coefficients of existing generators. In practice, the change in these coefficients should be translated to appropriate control of the injection of power sources associated with these generators. The control law is straightforward, $P_{add} = (m_F^* - m_0)\dot{\omega} + (d_F^* - d_0)\omega$. Such remedial actions would often require a high power low energy storage. Flywheel farms and superconducting magnetic energy storage (SMES) are expected to serve well such functions, along with other services they might offer. An additional comment here is about the communication delays. In this paper, we have assumed that at the occurence of the fault, inertia and damping are automatically tuned to the new values. In practice, there will be a delay from the moment the fault occurs till the moment the fault is detected and the new parameters set. We expect that all the tuned parameters for several different faults will be precomputed and stored locally at a lookup table. The power sources only need to receive a signal that a fault has occured to activate their control. This delay is expected to be in the range of tens of milliseconds. Future work will include the communication delays.

VIII. Conclusions

This paper introduced a novel approach to incorporate remedial actions in direct methods for transient stability assessment. Extending the work on quadratic lyapunov functions for robust stability and resiliency certificates, proposed in [9], we introduced inertia and damping control to enhance the transient stability of low-inertia systems. Future power systems will most probably have additional power sources to assist in the transient stability and compensate for the missing inertia of renewable generators. In effect such sources could mimic the inertia and damping coefficients of conventional generators. The goal of this paper is to incorporate the tuning of these sources in the robust certificates. By that we can guarantee system stability for a larger region of attraction, and eliminate the need to run time consuming time-domain simulations to examine system stability in these cases. An advantage of the proposed approach is that the parameters need to remain tuned only during the fault-on dynamics. Since the fault is cleared usually within some hundreds of milliseconds, the power and energy requirements for the additional power sources are low. A further contribution of this paper is the introduction of the rescaling factor Λ, which relaxes our linear matrix inequalities and allows us to obtain substantially better results. We described an algorithm to determine the optimal inertia and damping parameters to be tuned and presented a numerical example demonstrating the method.

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