Stochastic AC Optimal Power Flow with Approximate Chance-Constraints

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***Abstract*—With higher shares of fluctuating electricity generation from renewables, new operational planning methods to handle uncertainty from forecast errors and short-term fluctuations are required. In this paper, we formulate a probabilistic AC optimal power flow where the uncertainties are accounted for using chance constraints on line currents and voltage magnitudes. The chance constraints ensure that the probability of limit violations remain small, but require a tractable reformulation. To achieve this, an approximate, analytical reformulation of the chance constraints is developed based on linearization around the expected operating point and the assumption of normally distributed deviations. Further, an iterative solution approach is suggested, which allows for a straightforward adaption of the method based on any existing AC OPF implementation. We evaluate the performance of our method in a case study on the 24-bus IEEE RTS96 system. The proposed algorithm is found to converge fast and substantially reduce constraint violations.**

***Index Terms*—Chance Constrained Optimal Power Flow, AC Power Flow, Forecast Uncertainty**

I. Introduction

Due to the increased penetration of renewable energy sources (RES) such as wind and solar PV, Transmission System Operators (TSOs) have to cope with larger uncertainty in power system operation. This poses a variety of new challenges to the TSOs [1], such as how to efficiently handle congestion management and voltage variations. To ensure power system security and cost-optimal operation, the development of new tools that appropriately account for this uncertainty is necessary.

In this paper, we consider an extension of the AC Optimal Power Flow (AC OPF) to account for uncertainty in the power injections. While the traditional OPF formulation only considers a particular operating point, a number of methods to account for uncertainty around this operating point have been proposed. Some methods are used as an a-posteriori validation of the OPF results, such as the point-estimation method in [2], while others are actively accounting for uncertainty within the optimization, such as the robust approach in [3] or the two-stage, scenario-based approaches in [4], [5]. We will consider a third type of methods which directly involves uncertainty within the OPF, the so-called chance constrained OPF [6]–[13], which apply chance constraints to ensure that the probability of constraint violation remains small.

While chance constraints offer a comprehensive way of handling forecast uncertainty, it is generally challenging to reformulate the resulting OPF into a tractable optimization problem. Previous chance constrained OPFs were typically based on DC power flow, which are easier to handle due to the linear relation between power flows and power injections. Some formulations have however considered AC power flow, e.g. [7], [11]. In [11], the chance constrained problem is reformulated using a convex relaxation of the full AC power flow equations and a sample-based approach to handle the chance constraint. While this method guarantees feasibility of the chance constraints, computational tractability for large systems is still a challenge to be addressed. In [7], an iterative solution based on numerical integration and linearization of the AC power flow constraints is proposed, which leads to an approximate solution of the chance constrainted AC OPF.

In this paper, we propose a different iterative approximation for the chance constrained AC OPF. We express our AC constraints as the sum of the nominal values at the expected operating point (calculated based on the full AC equations) and the impact of the uncertain variables (which is calculated based on a linearization around the expected operating point). This chance constraint approximation was first applied to assess the probability of apparent power flow violations in [14], although without application in an optimization problem. The proposed linearization has several advantages. It allows for an analytical reformulation of the chance constraints [13], which makes it possible to account for uncertainty without adding additional variables or constraints to the AC OPF itself. Further, since the iterative solution approach only adds an outer iteration, the method can be implemented using existing AC OPF solvers. Although the accuracy of the method reduces as the error size increases, the introduced approximations provide good estimates for most chance constraint terms as observed in the case study.

The paper has three contributions. First, we develop a chance constrained AC OPF formulation. Extending existing work, we introduce approximated chance constraints on currents and voltages, and embed such constraints in an AC OPF setting for the first time. Second, the solution algorithm we propose is based on an iterative scheme which can use *any* AC OPF algorithm to solve the chance constrained problem. This can hopefully ease implementation in existing algorithms, currently in use by the utilities. Third, we assess the convergence of the method, and compare its performance to a Monte Carlo simulation.

The remainder of this paper is structured as follows. In Section II, we describe the AC OPF formulation with chance constraints and our approach to approximate the chance constraints, while Section III explains the iterative solution approach. In Section IV, a case study to assess the convergence and the performance of the method is presented. Section V summarizes and concludes.

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II. Optimal Power Flow Formulation

In this section, we first present the chance constrained AC OPF formulation, before we explain in more detail how the approximate reformulation is obtained.

A. AC Optimal Power Flow with Approximate Chance Constraints

To formulate the chance constrained AC OPF, we consider a power system with a set of conventional generators $\mathcal{G}$, a set of lines $\mathcal{L}$ and a set of buses $\mathcal{B}$. The electricity generation from wind power is assumed to be the only uncertainty source, although different uncertainties can also be incorporated within the considered framework. To secure the system against forecast uncertainty at lowest possible cost, we would like to solve the following chance-constrained AC OPF problem based on the full AC power flow equations:

$$\min_{\mathbf{P_G}} \quad \sum_{i \in \mathcal{G}} \left(\mathbf{c_{2}}_{,i} \mathbf{P}^{\mathbf{2}}_{\mathbf{G},i} + \mathbf{c_{1}}_{,i} \mathbf{P}_{\mathbf{G},i} + \mathbf{c_{0}}_{,i} \right) \tag{1}$$

$$\text{s.t.} \quad \mathbf{f}(\boldsymbol{\Theta}, \mathbf{V}, \mathbf{P}, \mathbf{Q}) = 0 \tag{2}$$

$$\mathbf{P}^{\mathbf{min}}_{\mathbf{G}} \leq \mathbf{P_G} \leq \mathbf{P}^{\mathbf{max}}_{\mathbf{G}} \tag{3}$$

$$\mathbf{Q}^{\mathbf{min}}_{\mathbf{G}} \leq \mathbf{Q_G} \leq \mathbf{Q}^{\mathbf{max}}_{\mathbf{G}} \tag{4}$$

$$\mathbb{P}(\tilde{\mathbf{I}}_{\mathbf{L},j} \leq \mathbf{I}^{\mathbf{max}}_{\mathbf{L},j}) \geq 1 - \epsilon, \quad \forall j \in \mathcal{L} \tag{5}$$

$$\mathbb{P}(\tilde{\mathbf{V}}_i \leq \mathbf{V}^{\mathbf{max}}_i) \geq 1 - \epsilon, \quad \forall i \in \mathcal{B} \tag{6}$$

$$\mathbb{P}(\tilde{\mathbf{V}}_i \geq \mathbf{V}^{\mathbf{min}}_i) \geq 1 - \epsilon, \quad \forall i \in \mathcal{B} \tag{7}$$

The objective (1) is to minimize the total cost of generation from conventional generators $\mathbf{P_G}$, with corresponding quadratic $\mathbf{c_2}$, linear $\mathbf{c_1}$ and constant $\mathbf{c_0}$ cost coefficients. Eq. (2) are the full non-linear AC equations for nodal power balance of active and reactive power, formulated for the nominal operating condition without any fluctuation. $\boldsymbol{\Theta}$, $\mathbf{V}$ are the nominal voltage angles and magnitudes per bus, and $\mathbf{P}$, $\mathbf{Q}$ denote the vector of nominal nodal active and reactive power injections, which are the sum of generation from conventional generators $\mathbf{P_G}, \mathbf{Q_G}$, forecasted generation from wind power plants $\mathbf{P_W}, \mathbf{Q_W}$ and loads $\mathbf{P_D}, \mathbf{Q_D}$ per bus:

$$\mathbf{P} = \mathbf{P_G} + \mathbf{P_W} - \mathbf{P_D}, \quad \mathbf{Q} = \mathbf{Q_G} + \mathbf{Q_W} - \mathbf{Q_D} \tag{8}$$

Eqs. (3), (4) ensure that the active and reactive power production of the generators stay within bounds. Eqs. (5) limit the magnitude of the transmission line currents $\tilde{\mathbf{I}}_{\mathbf{L}}$ to the upper limit $\mathbf{I}^{\mathbf{max}}_{\mathbf{L}}$, while (6), (7) ensure that the bus voltages $\tilde{\mathbf{V}}$ stay within bounds $\mathbf{V}^{\mathbf{min}}, \mathbf{V}^{\mathbf{max}}$. Since the magnitude of the currents and voltages depend on the wind power fluctuations, these constraints are modelled as *chance constraints*, i.e., we require that they will hold with a desired probability $1 - \epsilon$. Note that the power output of the generators will also change depending on the wind power production, due to reserve activation. However, the generator constraints (3), (4) are not modelled as chance constraints, since we assume that the wind power fluctuations are covered through preprocured reserve capacities (e.g. from the ancillary services market).

To solve the OPF problem, the chance constraints (5) - (7) must first be reformulated into tractable constraints. As shown in [13], [14], these constraints can be reformulated analytically if the constraints depend linearly on the uncertain variables $\boldsymbol{\Delta}\mathbf{P_W}$, $\boldsymbol{\Delta}\mathbf{Q_W}$. Note that non-linear dependence on other variables $\boldsymbol{\Theta}, \mathbf{V}, \mathbf{P}, \mathbf{Q}$ is not a problem for the reformulation.

We therefore reformulate the chance constraints by linearizing the system around the nominal operating point. This allows us to express the realized currents $\tilde{\mathbf{I}}$ and voltages $\tilde{\mathbf{V}}$ in (5) - (7) as the sum between the nominal currents $\mathbf{I}$ and voltages $\mathbf{V}$ (calculated using the full, nonlinear AC power flow equations) and an uncertainty term $\boldsymbol{\Delta}\mathbf{I}, \boldsymbol{\Delta}\mathbf{V}$ which depends linearly on $\boldsymbol{\Delta}\mathbf{P_W}, \boldsymbol{\Delta}\mathbf{Q_W}$. The analytical reformulation detailed in [13], [14] is then applied. We describe the reformulation procedure in more details in the subsequent sections.

B. Wind Power Modelling

We model fluctuations in the active wind power injections $\tilde{\mathbf{P}}_{\mathbf{W}}$ as the sum of the forecast $\mathbf{P_W}$ and a forecast error $\boldsymbol{\Delta}\mathbf{P_W}$. The forecast errors $\boldsymbol{\Delta}\mathbf{P_W}$ are assumed to follow a normal distribution with zero mean and covariance matrix $\boldsymbol{\Sigma}_{\mathbf{W}}$. We assume that the wind power plants are producing at a constant power factor $\cos\varphi$, such that

$$\tilde{\mathbf{P}}_{\mathbf{W}} = \mathbf{P_W} + \boldsymbol{\Delta}\mathbf{P_W} \ , \tag{9}$$

$$\tilde{\mathbf{Q}}_{\mathbf{W}} = \mathbf{Q_W} + \boldsymbol{\Delta}\mathbf{Q_W} = \rho(\mathbf{P_W} + \boldsymbol{\Delta}\mathbf{P_W}) \ , \tag{10}$$

where $\rho = \tan(\arccos(\cos\varphi))$. During wind power fluctuations, the active and reactive power injections of the conventional generators are also changing. The active power fluctuations are assumed to be balanced through the automatic generation control (AGC), where the generators are contributing according to their maximum outputs, similar to [10].

The reactive power fluctuations are not balanced centrally, but rather through adjustments in reactive power injections at the PV buses to keep the voltage constant. We assume that the change in reactive power generation at PQ buses is 0, except for buses where the wind power plants are connected. The adjustments in power production at each bus can be expressed through a matrix of generation shift factors $\boldsymbol{\Psi}_{\mathbf{P}}$ for active power and $\boldsymbol{\Psi}_{\mathbf{Q}}$ for reactive power, resulting in the generation shift matrix $\boldsymbol{\Psi}$:

$$\begin{bmatrix} \Delta\mathbf{P_{PV}} \\ \Delta\mathbf{P_{PQ}} \\ \Delta\mathbf{Q_{PQ}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Psi}_{\mathbf{P}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Psi}_{\mathbf{Q}} \end{bmatrix} \times \begin{bmatrix} \Delta\mathbf{P_W} \\ \Delta\mathbf{Q_W} \end{bmatrix} = \boldsymbol{\Psi} \begin{bmatrix} \Delta\mathbf{P_W} \\ \Delta\mathbf{Q_W} \end{bmatrix}$$

C. Voltage Constraints

The realized bus voltage magnitudes $\tilde{\mathbf{V}}$ are given by the sum of the nominal voltages $\mathbf{V}$ and the deviations $\boldsymbol{\Delta}\mathbf{V}$. As explained above, we use the full AC power flow equations to calculate the base case voltages $\mathbf{V}$ and then express the changes in the voltage magnitudes $\boldsymbol{\Delta}\mathbf{V}$ through linear sensitivity factors $\boldsymbol{\Gamma}_{\mathbf{V}}$, i.e.,

$$\tilde{\mathbf{V}} = \mathbf{V} + \Delta\mathbf{V} \approx \mathbf{V} + \boldsymbol{\Gamma}_{\mathbf{V}} \begin{bmatrix} \Delta\mathbf{P_W} \\ \Delta\mathbf{Q_W} \end{bmatrix} . \tag{11}$$

The sensitivity factors $\boldsymbol{\Gamma}_{\mathbf{V}}$ are derived based on the system Jacobians. The full Jacobian, with partial derivatives of the active and reactive power injections with respect to the voltage magnitudes and angles, can be expressed as

$$\begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix} = \begin{bmatrix} \frac{\partial\mathbf{P}}{\partial\mathbf{V}} & \frac{\partial\mathbf{P}}{\partial\boldsymbol{\Theta}} \\ \frac{\partial\mathbf{Q}}{\partial\mathbf{V}} & \frac{\partial\mathbf{Q}}{\partial\boldsymbol{\Theta}} \end{bmatrix} \cdot \begin{bmatrix} \Delta\mathbf{V} \\ \Delta\boldsymbol{\Theta} \end{bmatrix} = \mathbf{J}^{full}_{\mathbf{PQ}} \begin{bmatrix} \Delta\mathbf{V} \\ \Delta\boldsymbol{\Theta} \end{bmatrix}$$

Since the bus voltage of PV and VΘ buses as well as the angle of VΘ buses are fixed, the corresponding columns in $\mathbf{J}^{full}_{\mathbf{PQ}}$

can be omitted. In addition, we omit the rows corresponding to the unknown power injection (active and reactive power at the slack bus, reactive power at the PV buses), leading to a reduced, invertible matrix $\mathbf{J_{PQ}}$:

$$\begin{bmatrix} \Delta\mathbf{P_{PV}} \\ \Delta\mathbf{P_{PQ}} \\ \Delta\mathbf{Q_{PQ}} \end{bmatrix} = \mathbf{J_{PQ}} \times \begin{bmatrix} \Delta\mathbf{V_{PQ}} \\ \Delta\mathbf{\Theta_{PV}} \\ \Delta\mathbf{\Theta_{PQ}} \end{bmatrix} \tag{12}$$

By inverting equation (12) and adding zero-rows to $\mathbf{J_{PQ}^{-1}}$, the change in voltage magnitudes and angles as a function of the power injection fluctuations can be approximated through the following linear relation:

$$\begin{bmatrix} \Delta\mathbf{V} \\ \Delta\mathbf{\Theta} \end{bmatrix} = \mathbf{L} \times \begin{bmatrix} \Delta\mathbf{P_{PV}} \\ \Delta\mathbf{P_{PQ}} \\ \Delta\mathbf{Q_{PQ}} \end{bmatrix} = \begin{bmatrix} \mathbf{L_V} \\ \mathbf{L_\Theta} \end{bmatrix} \times \begin{bmatrix} \Delta\mathbf{P_{PV}} \\ \Delta\mathbf{P_{PQ}} \\ \Delta\mathbf{Q_{PQ}} \end{bmatrix}$$

where $\mathbf{L}$ is the expanded version of the inverse jacobian $\mathbf{J_{PQ}^{-1}}$, and $\mathbf{L_V}$, $\mathbf{L_\Theta}$ are the parts of $\mathbf{L}$ corresponding to magnitudes and angles, respectively. We can now express the sensitivity factors $\mathbf{\Gamma_V}$ for voltages as

$$\mathbf{\Gamma_V} = \mathbf{L_V}\mathbf{\Psi}. \tag{13}$$

Inserting (13) in (6), (7), we obtain the following voltage constraints for bus i:

$$\mathbb{P}\left(\mathbf{V}_i - \mathbf{\Gamma_V}_{(i,\cdot)} \times \begin{bmatrix} \Delta\mathbf{P_W} \\ \Delta\mathbf{Q_W} \end{bmatrix} \geq \mathbf{V}_i^{min}\right) \geq (1-\epsilon) \tag{14}$$

$$\mathbb{P}\left(\mathbf{V}_i + \mathbf{\Gamma_V}_{(i,\cdot)} \times \begin{bmatrix} \Delta\mathbf{P_W} \\ \Delta\mathbf{Q_W} \end{bmatrix} \leq \mathbf{V}_i^{max}\right) \geq (1-\epsilon) \tag{15}$$

With the linear dependency on $\Delta\mathbf{P_W}, \Delta\mathbf{Q_W}$, these constraints can be reformulated using the same approach as in [13], by assuming a normal distribution of both wind forecast errors and voltage deviation in PQ-buses. After reformulation, we obtain the following constraints

$$\mathbf{V}_i \geq \mathbf{V}_i^{\mathbf{min}} + \Phi^{-1}(1-\epsilon)\sqrt{\mathbf{\Gamma_V}_{(i,\cdot)}\mathbf{\Sigma_W}\mathbf{\Gamma_V}_{(i,\cdot)}^T} \tag{16}$$

$$\mathbf{V}_i \leq \mathbf{V}_i^{\mathbf{max}} - \Phi^{-1}(1-\epsilon)\sqrt{\mathbf{\Gamma_V}_{(i,\cdot)}\mathbf{\Sigma_W}\mathbf{\Gamma_V}_{(i,\cdot)}^T} \tag{17}$$

Here, $\Phi^{-1}(1-\epsilon)$ is the inverse cumulative distribution function of standard normal distribution, evaluated at the probability $1-\epsilon$. We observe that the uncertainty leads to a tightening of the respective constraints. This tightening is a constant and can be interpreted as a security margin against uncertainties, thus we call it an *uncertainty margin* for the bus voltages $\mathbf{\Omega_V}_i$:

$$\mathbf{\Omega_V}_i = \Phi^{-1}(1-\epsilon)\sqrt{\mathbf{\Gamma_V}_{(i,\cdot)}\mathbf{\Sigma_W}\mathbf{\Gamma_V}_{(i,\cdot)}^T} \tag{18}$$

D. Current constraints

Similar to the voltages, we approximate the changes in the line current magnitudes through linear sensitivity factors $\mathbf{\Gamma_I}$:

$$\tilde{\mathbf{I}}_\mathbf{L} = \mathbf{I_L} + \mathbf{\Delta I_L} \approx \mathbf{I_L} + \mathbf{\Gamma_I}\begin{bmatrix} \Delta\mathbf{P_W} \\ \Delta\mathbf{Q_W} \end{bmatrix}. \tag{19}$$

More specifically, we approximate $\mathbf{\Delta I_L}$ as

$$\mathbf{\Delta I_L} \approx \begin{bmatrix} \frac{\partial \mathbf{I_L}}{\partial \mathbf{V}} & \frac{\partial \mathbf{I_L}}{\partial \mathbf{\Theta}} \end{bmatrix} \times \begin{bmatrix} \Delta\mathbf{V} \\ \Delta\mathbf{\Theta} \end{bmatrix} = \mathbf{J_I} \times \begin{bmatrix} \Delta\mathbf{V} \\ \Delta\mathbf{\Theta} \end{bmatrix} \tag{20}$$

By combining (20) with (13), the current sensitivity factors ($\mathbf{\Gamma_I}$) can be expressed as:

$$\mathbf{\Gamma_I} = \mathbf{J_I} \times \mathbf{L} \times \mathbf{\Psi} \tag{21}$$

As for the voltage constraints, we use the sensitivity factor to express (5) for a particular line j as:

$$\mathbb{P}\left(\mathbf{I_L}_j + \mathbf{\Gamma_I}_{(j,\cdot)} \times \begin{bmatrix} \Delta\mathbf{P_W} \\ \Delta\mathbf{Q_W} \end{bmatrix} \leq \mathbf{I_L}_j^{max}\right) \geq (1-\epsilon) \tag{22}$$

Applying the same type of reformulation as above, the constraint reduces to

$$\mathbf{I_L}_j \leq \mathbf{I_L}_j^{max} - \Phi^{-1}(1-\epsilon)\sqrt{\mathbf{\Gamma_I}_{(j,\cdot)}\mathbf{\Sigma_W}\mathbf{\Gamma_I}_{(j,\cdot)}^T} \tag{23}$$

Finally, the uncertainty margins for current magnitudes can be formulated as:

$$\mathbf{\Omega_I}_j = \Phi^{-1}(1-\epsilon)\sqrt{\mathbf{\Gamma_I}_{(j,\cdot)}\mathbf{\Sigma_W}\mathbf{\Gamma_I}_{(j,\cdot)}^T} \tag{24}$$

III. Iterative Solution Algorithm

To solve the chance constrained AC OPF, we replace the chance constraints (5) - (6) by their approximations,

$$\mathbf{I_L} \leq \mathbf{I_L^{max}} - \mathbf{\Omega_I}, \tag{25}$$

$$\mathbf{V} \leq \mathbf{V^{max}} - \mathbf{\Omega_V}, \tag{26}$$

$$\mathbf{V} \geq \mathbf{V^{min}} + \mathbf{\Omega_V}. \tag{27}$$

However, since $\mathbf{\Omega_V^k}, \mathbf{\Omega_I^k}$ are computed based on the Jacobians, they depend on the operating point of the system. Therefore, we solve the problem using an iterative approach, which uses the maximum change of the uncertainty margins (between subsequent iterations) as a measure of convergence. The approach consist of the following steps:

1) *Initialization:* Set uncertainty margins $\mathbf{\Omega_V^0} = \mathbf{\Omega_I^0} = 0$, and iteration count $k = 0$.
2) *Solve AC OPF:* Solve the AC OPF defined by (1) - (4), (25) - (27), and increase iteration count $k = k + 1$.
3) *Evaluate uncertainty margins:* Compute the uncertainty margins of the current iteration $\mathbf{\Omega_V^k}, \mathbf{\Omega_I^k}$, and evaluate the maximum difference to the last iteration for voltages $\delta_V^k = \max\{|\mathbf{\Omega_V^k} - \mathbf{\Omega_V^{k-1}}|\}$ and currents $\delta_I^k = \max\{|\mathbf{\Omega_I^k} - \mathbf{\Omega_I^{k-1}}|\}$.
4) *Check convergence:* If $\delta_I^k \leq \hat{\delta}_I^k$ and $\delta_V^k \leq \hat{\delta}_V^k$, stop. If not, move back to step 1.

The accuracy of the solution is defined through the choice of the stopping criteria $\hat{\delta}_I^k, \hat{\delta}_V^k$.

IV. Case Study

To evaluate the performance of the proposed OPF method, a case study based on the IEEE RTS96 system with 24 buses [15] was performed. We assume wind power generation on bus 8 and 15, with forecasted power production of 250 and 350 MW, respectively. The forecast errors are modeled as zero mean, multivariate Gaussian random variables with a standard deviation corresponding to 20% of the forecasted production. For reactive power generation, we assume that the wind power plants are operating with a constant power factor $cos\varphi = 0.95$ capacitive. The acceptable violation probability is set to $\varepsilon = 0.05$. The cost functions of the conventional generators are modeled based on the generator cost coefficients provided with Matpower [16].

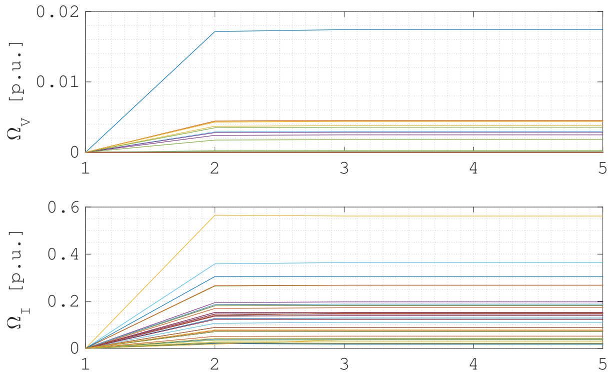

Fig. 1. Values of the uncertainty margins for each iteration until convergence. Each line represents one uncertainty margin. Top: Voltage uncertainty margins for each bus of the system. Bottom: Current uncertainty margins for each line in the system.

A. Convergence of the probabilistic AC OPF

In the following, we apply the iterative solution scheme described in Section III to solve the chance constrained AC OPF. The chance constrained AC OPF was implemented based on the standard AC OPF given in [17], using the SNOPT solver from Tomlab Optimization. This was adapted to include an iterative update of the uncertainty margins and inequality constraints on currents. As a stopping criterion, we define $\hat{\delta}_I^k = 10^{-5}\, p.u.$ for currents and $\hat{\delta}_V^k = 10^{-6}\, p.u.$ for voltages. The convergence of the uncertainty margins is depicted in Fig. 1. It can be seen that the constraint tightenings are converging quickly. For the chosen operating point and convergence tolerance, only five iterations are necessary.

B. Accuracy of the Uncertainty Margins

After solving the chance constrained AC OPF, we evaluate the accuracy of the approximated uncertainty margins. To this end, we use a Monte Carlo simulation with 10000 samples of the forecast uncertainty drawn from a multivariate Gaussian distribution. For each sample, we solve the full AC power flow using Matpower [16] and record the bus voltages and line currents. Based on the results, we estimate the true upper and lower voltage uncertainty margins $\mathbf{\Omega_V^+}, \mathbf{\Omega_V^-}$ as the empirical 5% and 95% quantiles, respectively. Note that while the approximated voltage uncertainty margin (18) are the same in both directions, the upper and lower margins are not necessarily the same, since the full AC power flow equations depend non-linearly on the change in wind power production.

A larger difference in the upper and lower uncertainty margins $\mathbf{\Omega_V^+}, \mathbf{\Omega_V^-}$ indicate that linearization is a less appropriate approximation. To estimate the effect of nonlinearlities, we also define an upper and lower uncertainty margins $\mathbf{\Omega_I^+}, \mathbf{\Omega_I^-}$ for the line currents, although only the upper margin $\mathbf{\Omega_I^+}$ is of practical interest.

The uncertainty margins for the bus voltages are shown in Fig. 2. We observe that the approximate uncertainty margins always lie between the upper and lower margin. The approximated margin $\mathbf{\Omega_V}$ is larger than $\mathbf{\Omega_V^+}$ and smaller than $\mathbf{\Omega_V^-}$ for all buses, implying that the upper voltage constraints would have violation probabilities $< \epsilon$ while the lower voltage constraints might be violated with violation probabilities $> \epsilon$.

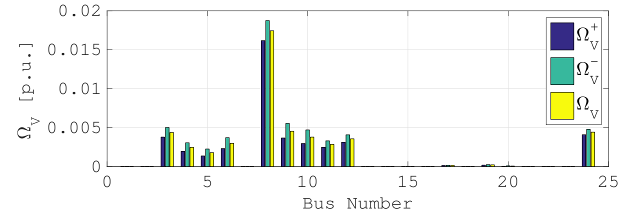

Fig. 2. Voltage uncertainty margins obtained from the Monte Carlo simulation $\mathbf{\Omega_V^+}, \mathbf{\Omega_V^-}$ and through the approximation $\mathbf{\Omega_V}$.

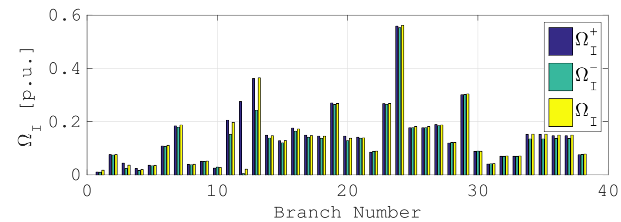

Fig. 3. Current uncertainty margins obtained from the Monte Carlo simulation $\mathbf{\Omega_I^+}, \mathbf{\Omega_I^-}$ and through the approximation $\mathbf{\Omega_I}$.

The uncertainty margins for the line currents are shown in Fig. 3. Compared to the voltage margins, the current margins are much larger in absolute terms, with most values ranging from 0.1 - 0.5 p.u. The approximated uncertainty margins are close to the margins estimated from the Monte Carlo simulation. The upper and lower Monte Carlo margins have similar values for most lines, indicating that the linearized uncertainty margins are a good approximation. However, for lines 11, 12 and 13, the difference between the upper and lower margins are larger, e.g., > 0.25 p.u. for line 12. To analyze why this happens, we investigate the current magnitudes for the three lines and the "normal" line 10 in more detail. In Fig. 4, the current magnitudes obtained from the Monte Carlo simulation are plotted against the active wind power production at bus 8. We observe that for line 10, the current is increasing linearly with the active power production, whereas for lines 11, 12 and 13, the current first decreases and then increases again, due to a change in the direction of the active power flow. This implies that, despite the poor estimation of the current uncertainty margin on the lines where the power flow changes direction, the suggested method still works well, as the current magnitudes in the corresponding lines are rather small compared to the long term current rating of $1.75\, p.u.$

C. Comparison of OPF to chance constrained OPF

To conclude the case study, we compare the solution of the chance constrained AC OPF with the solution of the

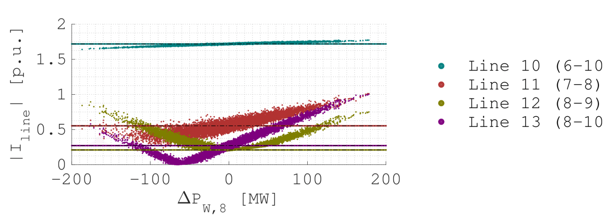

Fig. 4. Current magnitudes from the Monte Carlo simulation, plotted against the wind power deviation at bus 8.

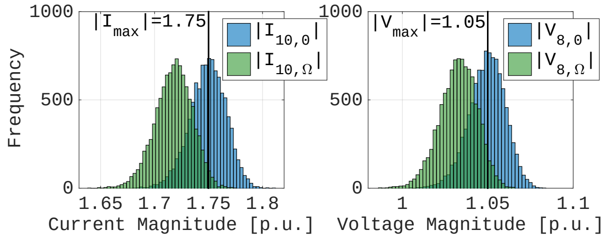

Fig. 5. Histogram of the Monte Carlo results for the standard AC OPF (subscript 0) and the chance-constrained AC OPF with uncertainty margins (subscript Ω). Left: Current magnitudes in line 10. Right: Voltage magnitudes at bus 8.

standard AC OPF (without uncertainty margins). In both cases, the current magnitude constraints on line 10 and the voltage constraint at bus 8 were active. Again, Monte Carlo simulations with 10000 samples were run for each of the two solutions, and the results were used to produce a histogram of the current and voltage magnitudes for the respective solutions, as shown in Fig. 5.

On the left side of Fig 5, the histograms of the current magnitudes of line 10 are shown. The vertical line at $1.75\,p.u.$ represents the line limit. One can see that the optimal operating point determined by the AC OPF leads to around $50\,\%$ probability of constraint violation, while chance constrained AC OPF reduces the violation probability to about $2\,\%$ (i.e., to below the acceptable violation probability $\epsilon = 5\%$).

On the right side of Fig 5, the histogram of the voltage magnitudes are shown. Again, the standard AC OPF leads to around 50% violation probability. The chance constrained AC OPF reduces the probability to about 4%, which is again below the acceptable probability $\epsilon = 5\%$.

While the chance constrained AC OPF increases security, the uncertainty margins also make the problem more constrained and thus increase cost. In the example shown here, the tightening on the relevant line currents and bus voltages is relatively small. The cost increases only minimally, from 48463 $/h with the standard AC OPF to 48494 $/h with the chance constrained AC OPF.

V. Conclusions

This paper proposes a method to account for forecast uncertainty from renewables in an AC Optimal Power Flow. We first formulate the OPF as a chance constrained optimization problem, and then propose an approximate reformulation of the chance constraints based on linear sensitivity factors and analytical relations. Further, an iterative solution approach where the sensitivity factors are updated in each iteration after solving the AC OPF, is proposed.

The convergence of the iterative scheme as well as the performance of the chance constrained approach was investigated in a case study based on the IEEE RTS96 system with added wind power generation. The iterative scheme was found to converge fast, and the reformulation appears to be working well. The probability of constraint violation was substantially reduced compared to a standard AC OPF without any consideration of uncertainty.

In future work, we plan to investigate if the method, which in this paper was based on normally distributed forecast errors, can benefit from less restrictive assumptions on the distributions [13]. Moreover, we would like to investigate other solution approaches, including an internal update of the uncertainty margins, which would allow for optimization of e.g. reserve activation along the lines of [12]. Finally, the formulation should also be developed further to better account for the influence of uncertainty on generator output, which would allow for co-optimization of energy and reserves, as well as security constraints to ensure N-1 security.

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